SIMULATION OF TRAFFIC CONGESTION AT UNSIGNALISED INTERSECTIONS USING A MICROSCOPIC TRAFFIC FLOW MODEL

MAITHYA DENNIS MAMBO
MC300-0002/2015

A Research submitted to Pan African University Institute for Basic Sciences, Technology and Innovation in partial fulfillment of the requirement for the award of the degree of Master of Science in Mathematics (Computational Option) of the Pan African University.

2017

DECLARATION

I declare that this work is my original work and that it has never been submitted to any institution of higher learning for any award or assessment. Where information from others sources have been used, the citation has been made.

Signature: ....   Date: ....

Maithya Dennis Mambo

MC300-0002/2015

This work has been carried out under our supervision as the university supervisors..

Signature: ....   Date: ....

Dr. Mark Kimathi

Faculty of Applied Sciences and Technology, The Technical University of Kenya, NAIROBI.

Signature: ....   Date: ....

Prof. Diaraf Seck

Department of Mathematics of Decision, Laboratory of Mathematics Decisions and Numerical Analysis, University of Chiek Anta Diop of Dakar, Dakar-Fann, Senegal.

DEDICATION

This work is dedicated to God Almighty.

ACKNOWLEDGEMENT

LIST OF FIGURES

NOMENCLATURE (NOTATIONS)

i: the vehicle

$x_i(t)$: the position of the front bumper of the following vehicle with respect to time

$x_{i+1}(t)$: the position of the front bumper of the preceding vehicle with respect to time

$v_i(t)$: the velocity of the following vehicle i at time t

$v_{i+1}(t)$: the velocity of the preceding vehicle $(i + 1)$ at time t

$x_{i+1} - x_i$: the spacing between the preceding vehicle $(i + 1)$ and the following vehicle (i) at time t

$v_{i+1} - v_i$: the velocity difference between vehicle $(i + 1)$ and vehicle (i) at time t

l_i: is the spacing between the preceding vehicle and the following vehicle

H: is the length of the vehicle (considered as a constant here)

$U(\rho_i)$: is the equilibrium velocity

T: Reaction Time. It is the time a vehicle takes to adjust to the equilibrium velocity.

ΔT: is the interval of time required for a driver to react to a changing situation. It is the time the ith vehicle takes to move from one point to the next in response to the stimuli induced.

C: is a constant. It scales the anticipation term

LIST OF ABBREVIATIONS

MITSIM: Microscopic Traffic Simulator.

MLC: Mandatory Lane Changes.

DLC: Discretionary Lane Changes.

GM: General Motors Microscopic Car Following Model.

SITRAS: Massive Multi-Simulation System.

MATLAB: A Mathematical Programming Platform.

C# : A Computing Programming Software Package.

ABSTRACT

Traffic flow in most urban areas is augmenting due to growth in transport and continual demand for it, most particularly at unsignalised intersections. To address this, we developed and analyzed a Microscopic Traffic Flow Model for describing traffic flow at those unsignalised intersections that incorporated lane-changing maneuvers and performed a simulation of the traffic formation. A Microscopic Traffic flow model describes the motion of each vehicle by modeling the action of the vehicles such as the acceleration and deceleration of each driver. A qualitative study was done. We proved that intersections can cause traffic jams if not well managed. To achieve this, the general motors car following model was used so that the motion of each vehicle was discretely monitored and this discretization was done using the Forward Finite Difference Method. The existence and uniqueness of the results was done by invoking the lipschitz conditions. The results obtained for example the back-chaining of the leading vehicles behaviour was replicated in the traffic scenario considered and agreed with what happen in real world.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

This study focuses on unsignalised intersections. In most cases from here going forward, the word intersection wherever it appears will be used to refer to an unsignalised intersection. It is arguably correct to say that an efficient transport system is essential for the optimal functioning and prosperity of any modern economy. The quality of life, self-fulfillment, and personal freedom to some extent depend on mobility as a key catalyst to this satisfaction. In achieving the mobility, this study proposes a mathematical traffic simulation model using the General (mostly, from here on, the word intersection will be used to refer to unsignalised intersection)Motors microscopic car-following model to simulate traffic at these intersections.

The traffic simulation model has been chosen because it adequately and extensively represents real life and applies different strategies without the need to make any physical change on the site before implementing such strategies. Again, simulation modeling is an increasingly popular and effective tool for analyzing a wide variety of dynamical problems especially those associated with complicated processes, which are not readily describable in analytical terms. Usually, these processes are can be described by the interaction of many system components or entities whose interactions are complex in nature.

Car-following models represent the basic unit that governs the longitudinal movement for each traffic simulation model. The efficiency of a traffic simulation model mainly depends on its core units: car-following and lane changing. Also, we shall consider different traffic conditions such as high speed, low speed, "stop, and go conditions."

1.1   Background of Study

The proliferate development and the tremendous growth in the population of most cities has had a significant influence on the travel pattern(s) of the community from one place to another. This results from an increased transport demand which in most cases leads to a rise in the cost of living of the citizenry (Ruskin and Wang, 2002). The transportation system is also affected by the annual increase of the vehicles on the road.

For example, in 2014, Clement et al. (2014) in the Bloomberg Business magazine reported that the number of cars on Nairobi's roads has doubled to 700,000 since 2012 this however, has not been matched with infrastructure and traffic management. Again, Bloomberg reported that there is a foreseeable increase in the number of these vehicles to about 9 million by 2050 (Clement et al., 2014). With this growth in the number of vehicles on the roads, it naturally increases road congestion, especially during peak hours. Furthermore, it's worthwhile to note that the road network itself also influences traffic congestion. In a road network, the intersection is one of the leading bottlenecks causing traffic congestion. In developing economies, traffic and transportation is very different from that in developed countries since traffic composition and level of roadside activities differ.

In developing economies, traffic rules, for example, to give way or lane discipline etc., are neglected in most cases. Drivers are more aggressive so that a gap acceptance behavior is rather uncommon. Further, vehicles types in developing countries show a large variety, which makes traffic flow rather heterogeneous. This traffic flow consists of transport models of varying dynamic characteristics sharing the same road space. In this view, vehicles contribute to variation in speed behavior ranging from slow vehicles to rather fast-moving cars. Typical for developing countries, there is also a great number of activities occurring at the

edge of the road, both on the roadway shoulders and sidewalks. Most of these activities create a number of conflicts called *side frictions.*

The Indonesian manual gives much attention to side frictions such as aspects like pedestrians, stops by transport vehicle and parking manoeuvres, motor vehicle entries and exits into and out of roadside properties and side roads. According to this manual, they have great impact on capacity and performance (Prasetijo, 2007).

The unsignalised intersection is a common type of intersection used in the control of traffic movements. It plays a significant role in determining the overall capacity of road networks. For example, a poorly operating unsignalised intersection may affect nearby/neighboring signalized intersections. This means that the design of the intersection should be done appropriately considering all road factors necessary in the development of a road in order that avoidable traffic congestion is exempted.

Unsignalised intersections are the most common intersection types for understanding driver behavior and patterns of the inter-arrival times for vehicles and other properties such as capacities on the roads. Understanding the theory of the operation of unsignalised intersections is fundamental to many elements used for design and operation of other intersections. An analysis procedure for the road conditions is needed in the design of the unsignalised intersection so that capacity is always greater than traffic demand.

The traffic flow of a movement at an unsignalised intersection is guided by the hierarchical position of the movement specified either tacit (by rules of driving) or decreed (through static signs, such as "STOP" or YIELD').

At any unsignalised intersection, there are various types of movements, like;

- Through movement on major street,

- Right turn movement from major streets,

- Left turn movement from major streets,

- Through movement of minor street.

This study begins with a simple T-junction. The assumption here is that vehicles on the major road have a higher priority over those on minor roads/streets. In the figure below, $ABCD$ represents vehicles moving along the major road while EF represents vehicles moving along a minor road.

Figure 1.1:   A T-Junction Intersection
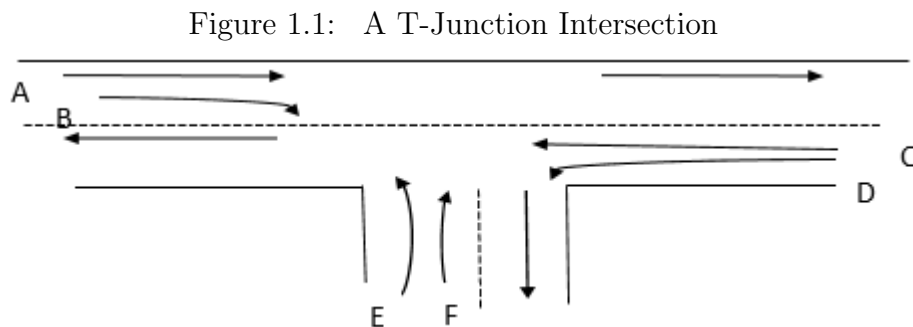
Each of the movements in Figure 1.1 has a place in the hierarchy specifying their claim on the right-of-way at the common intersecting space. In general, first in the hierarchy is the through movement on the Major Street; second is the right turn from a Major Street. For example, if in a situation there is a vehicle on the right turn movement and another on the conflicting through movement, the latter uses the intersection first while the former waits until the latter clears the intersection. If some other movement is still lower down the hierarchy for example the right turn from Minor Street, then a vehicle for that movement has to wait until the vehicles on the movements higher up in the hierarchy has cleared the intersection.

In Figure 1.1, A and C are entirely independent. For F to enter the main road it depends on A, B and C while for E to enter the major road, it depends on C. Finally, for B to enter the minor road, it depends on C (independence here means right of way while dependency means the vehicle has to wait till the intersection is clear).

This study builds up from the scenario described above. The scope of this study is to develop a mathematical model for describing flow of traffic at unsignalised intersections and perform simulations to show how congestion occurs.

The motivation behind this research revolves around the desire to address the need expressed by transportation professionals for a better understanding of complicated traffic behaviour at highway intersections and the relationships that lie behind this behaviour.

Secondly, the need to explore, understand and effectively use the significant developments in computer technology and traffic simulation which promises potential development of new improved and robust analytical techniques. The complexity of traffic stream behaviour and the difficulties in performing experiments with real world traffic make computer simulation an important analysis tool in traffic modeling and engineering.

1.2   Statement of the Problem

Traffic congestion is least expected to end in the near future at road intersections because of the ever increasing transport growth and continuous demand for traffic flow (Raslaviius et al., 2015). Traffic congestion is a major problem of transport in many countries (Huang and Sadek, 2009; Chen et al., 2012). The unstoppable increasing urbanization over the years has resulted in the development of vast areas as urban extensions. In some urban centers, many roads were laid in an incremental manner to cater to the increased traffic demand (Goerigk et al., 2013;

Lo et al., 2011).

The cities have developed in a disintegrated urban form spreading along major traffic corridors (Raslaviius et al., 2015). Congestion on many of these roads is due to haphazard development, narrow streets, congested junctions, unorganized parking etc. which creates hindrance to the smooth flow of traffic (Raslaviius et al., 2015). Most of the major intersections are no longer able to cope with even the present traffic demand. The cause of this results from bottlenecks for example merging and intersections. Unsignalised intersections are examples of bottlenecks that are interesting in the study of traffic congestion since they are uncontrolled (Zhang and Kim, 2005). The way vehicles enter their lane of choice without much regard to the other drivers makes congestion grow spreading far and wide. High travel time and congestion have created an adverse effect on the economic and environmental health of the cities (Raslaviius et al., 2015).

Zhang (2004) describes microscopic (micro) simulation models as more and more widely used to support real-time control and management functions in the field of transportation planning, functional design, and operation engineering. Car following models predict the response of the following vehicle to the stimulus caused by the lead vehicle (Zhang, 2004). Improper understanding and consideration of all traffic scenarios/traffic characteristics at intersections leads to inaccurate interpretations of the results of the analysis which are key in making recommendations about the design and location of the intersection.

We seek to address the problem of traffic congestion at unsignalised intersections which from deep study reveals that it is one of the biggest causes of traffic congestion in many cities in most African cities. The congestion is so dire that it affects it negatively affects the economic growth of these cities with reference to the amount of time lost in these traffic jams.

A good understanding ensures that all the parameters and variables chosen for the study are well evaluated. Planning improvements and resource mobilization

to expand the roads are quite expensive and unsustainable hence, cannot be relied upon. Therefore, it is very important to redevelop procedures for integrating various local traffic characteristics for a thorough analysis. The GM model chosen is extended to include lane changing manoeuvres which form the basis for describing traffic flow at the unsignalised intersections. The study work is focused on the General Motors car-following model that is of particular importance because it's comprehensive.

1.3 Justification of the study

Since traffic modeling is one of the most effective strategies in decongesting traffic jams, the development of a model that would predict and solve/reduce traffic congestion in real time is significant. The study seeks to develop a mathematical model for describing traffic flow at unsignalised intersections and later perform simulations of the traffic at those intersections. This study concentrates on extending the basic General Motors Car-Following Model by a mathematical investigation of traffic at these intersections.

1.4 Objectives of the study

1.4.1 General Objective

The general objective is to develop a microscopic traffic flow model for describing traffic flow at unsignalised intersections and then, to simulate the formation of traffic congestion at those unsignalised intersections.

1.4.2 Specific Objectives

(i) To extend the basic General Motors microscopic car-following model to incorporate lane-changing (turning) manoeuvres that happen at the intersections.

(ii) To use the extended microscopic model to investigate the formation of traffic congestion at the unsignalised intersections T-Junction(s).

(iii) To determine the appropriate strategies for merging vehicles at unsignalised intersection T-Junction(s).

1.5   Significance of the study

This study is significant for the following reasons.

(i) The study will assist the transport sector to optimize intersections consequently reducing the travel times wasted because of traffic build-ups.

(ii) It will contribute to improving control strategies used in reducing the occurrence of congestion at the intersection(s). This can be extended to other intersections with similar characteristics.

(iii) Through this study, public transport policy makers will be guided by optimal control strategies, which can be employed to control and mitigate traffic build-ups.

(iv) The study will be used to inform the national transport authorities about the burden of traffic jams and the measures to undertake to reduce the traffic.

(v) The study will help transport administrators to develop educational seminars, workshops or training programs to educate transport stakeholders and other interested parties about the importance of traffic rules observance and the need to participate in all the policy making processes and the traffic decisions undertaken.

CHAPTER 2

LITERATURE REVIEW

A number of studies have been conducted to highlight the plight of the intervention strategies and control of traffic congestion. In this chapter, a literature review of the acceleration and lane changing models is presented. Findings from this review are summarized at the end of the chapter.

2.1   Introduction

There are three types of modeling the traffic flow namely; microscopic, mesoscopic (between micro- and macro-) and macroscopic modeling. A Mecroscopic model blends the properties of both microscopic (discussed below) and macroscopic simulation models. Therefore, a mecroscopic model provides less fidelity than micro-simulation tools, but is superior to the typical planning analysis techniques (Kerner, 2009).

Macroscopic simulation modeling is based on the deterministic relationships of the flow, speed, and density of the traffic stream.

The simulation in a macroscopic model does not take place by considering or tracking individual cars but rather by considering section-by-section of the traffic stream.

Originally, Macroscopic simulation models were developed to model traffic in definite transportation sub-networks, such as freeways, corridors (including freeways and parallel arterials), surface-street grid networks, and rural highways. Microscopic simulation models simulate the movement of individual vehicles based on car-following and lane-changing theories (Kerner, 2009).

These models are effective in evaluating heavily congested conditions, complex

geometric configurations, and system-level impacts of proposed transportation improvements that are beyond the limitations of other tool types. However, these models are time consuming, costly, and can be difficult to calibrate. It's from this significant advantage of the Microscopic Model that it forms the basis for this research in this study.

Microscopic modeling can be said to start generally with, and focus on individual car movement. Systematics (2005) correctly puts it that most microscopic models can be referred to as "Headway models" because the single car movement relates to the headway between the two cars (or more) under consideration. Due to the interactions of cars at intersections, these microscopic models can also be called "Interacting models," since for intersections or roundabouts, for example, individual car movement may be inter-dependent.

2.2   Car Following Theory and Car-following Models

According to Gazis et al. (1961) and Robert and B (1961), the first and most basic microscopic models were those based on follow-the-leader theory also called "car following" theory. In this theory, the individual motion is essentially a reaction to the behaviour of the vehicle (the leading vehicle) in front and car movement is restrained by other conditions such as engine power, delay times and traffic rules. The model gives a stimulus-response function of the headway distance between the leading vehicle and the following vehicle, the speed of the following vehicle and their relative speed. Many experiments have been conducted over the years on microscopic modeling. Available literature shows that an actual car-following behaviour has several important features, which have not been explored by car following theory.

These include;

(i) Car following behaviour is "human behaviour", a process characterized by "vagueness" rather than determinism.

(ii) Response to stimuli in car following is asymmetric.

This contrasts the theory, which requires acceleration and deceleration to be symmetric. It is suggested that this is because "drivers pay closer attention due to a decrease in spacing (decrements) than to increase in spacing (increments) simply for their safety." The theory also assumes that the desired speed depends on the gap between vehicles. Accordingly, only one platoon (or set of cars with no intermediate spaces) will exist if the time of consideration is long enough.

This assumption is only correct when the speeds of the cars are less than the desired speeds of the drivers. The desired driving speeds vary considerably between the drivers, and depend not only on gaps between the vehicles in front but also on personal preferences, motivations for the journey, weather, car performance and road situations, the existence of several platoons is normal.

For instance, on a single-lane motorway, there are always several leader cars that have the lowest preferred speed in this platoon. Car-following theories describe how a vehicle follows another vehicle in continuous (an uninterrupted) flow. In all microscopic simulation models, the car-following process remains important as it is in modem traffic flow theory (McDonald et al., 1997). Additionally, according to Delis et al. (2014), traffic flow modeling and simulation have attracted a rapidly growing interest over the past years because of the following reasons;

- The need for optimization of the usage of existing traffic infrastructure and the design of new structures,

- The growing economic /environmental cost of traffic jams, their impact on the quality of life, the need to assess and optimize new driver assistance

devices, and

- The existence of complicated non-linear dynamical phenomena associated with traffic flow and the characteristics of vehicles and drivers.

The effective modeling of such phenomena, like traffic jams, stop-and-go traffic and "synchronized" congested traffic (Helbing et al., 2002), call for the development of an accurate theoretical model. These reproduce, at least qualitatively, all traffic states with the minimum number of parameters and the corresponding computational procedures, which should be numerically efficient (regarding computational time and memory requirements), accurate and robust, to allow for their use in real-world simulation and optimization scenarios (Delis et al., 2014). Several approaches have been followed over the past years to model the dynamics of traffic flow, the most successful ones being microscopic car-following models and macroscopic traffic models. The primary focus here will revolve around the Microscopic Car Following Model.

Microscopic simulation models have proven to be very useful and widely accepted tools for the analysis and management of transportation systems (Ntousakis et al., 2014). Within these tools, lane-changing and merging models belong to the most complicated and critical ones Kolen (2013); Lane-changing and merging are identified as significant sources of collisions and congestion on freeways under critical or heavy traffic conditions, while driver's psychological components have multiple dimensions that affect freeway merge decisions (Wang, 2005; Kou and Machemehl, 1997).

Most micro-simulation models are based on a gap-acceptance, combined with a car-following model Kolen (2013). The gap acceptance theory implies that every driver has a critical gap to complete lane changing safely Loot (2009). More specifically, each driver decides whether to accept or reject the available gap on the shoulder lane by comparing it to the critical gap (minimum acceptable

gap). The first framework for modeling the structure of lane changing decisions was developed by Gipps (1986). His model leads to the decision whether it is physically possible, necessary and safe to change lanes, taking into account parameters such as traffic signals, obstructions, the presence of transit lanes and heavy vehicles, the speed, etc. the micro-simulation model proposed by Hunt and Yousif (1990) for the merging behaviour at road networks was based on rules similar to those of the Gipps model. Yang and Koutsopoulos (1996), presented the Microscopic Traffic Simulator (MITSIM) package, which uses discrete choices for modeling lane changing behaviour in combination with a gap acceptance model. Based on Gipps theory, the microscopic traffic simulator (CORSIM) distinguishes lane changes as mandatory (MLC) or discretionary (DLC). MLCs are performed when the driver must shift from the current path to another (e.g. to use an off-ramp) to follow his route, whereas DLCs are performed when the driver changes lane just to improve his driving conditions (e.g. to overtake a slower vehicle). Ahmed (1999) developed a lane-changing model and an acceleration model to describe merging behaviour under heavily congested traffic. Moreover, the Hidas (2002) and Hidas and Behbahanizadeh (1999) developed a massive multi-agent simulation system (SITRAS) in which a lane changing and merging algorithm was incorporated, based on Gipps" model and additionally taking into account the concept of "forced merging" and "courtesy yielding" the so-called cooperative merging. In fact, automated merging systems had been a research topic as early as the 1960 (Luspay et al., 2010) and the reference therein.

More recently, Wang (2005) proposed a framework for freeway merging by combining acceleration and a gap-acceptance model. They considered the cooperativeness of the vehicles on the main road introducing the cooperative lane changing (in order to allow vehicles to merge) and courtesy yielding accretion (deceleration in order to create gaps). Additionally, Hidas (2005) developed a merging model to examine the cooperative behaviour of drivers introducing ex-

plicit model, which has four levels of decision-making : normal gap acceptance, the decision onto initiate, forced merging and the gap acceptance for courtesy and forced merging.

In Loot (2009), a new framework for modeling merging behaviour is proposed based on the gap selection, instead of the usual gap acceptance theories in combination with the cooperative behaviour of vehicles on the main road. Specifically, he points out that gap acceptance models, where the driver changes lane if the gap is large enough, do not respond to the actual merging behaviour. With the proposed model, every merging vehicle can find a suitable gap without being overtaken by many vehicles on the main road and without coming to standstill at the end of the on-ramp.

In recent years, the development of automation application by car manufactures and potential contribution of such systems to the merging behaviour of the equipment vehicles have further increased the interest of researchers on the subject.

Lu et al. (2000) presented a real-time implementation of a longitudinal control algorithm for vehicle merging into automated highway systems, proposing a concept of virtual platooning. Furthermore, Chen et al. (2012) (as cited in Delis et al., 2014) developed a detailed model for simulating merging situations in one-lane with dedicated through lane and entrance ramps and assuming that vehicles on this one-lane are operated fully automatically. The gaps are created on the main lane, lane following orders obtained by the infrastructure. Only if there is an available gap, the vehicle moves on; otherwise, it stops on the ramp, waiting for the next open gap.

Going forward, a comprehensive study on the General motors is given. The most well-known General motors car-following models were first developed by Chandler et al. (1958) from a simple as a starting point with an assumption that the response (acceleration or deceleration) of the following vehicle is proportional to the stimuli (relative velocity) between the lead and following vehicles. Then a

series of field experiments were carried out on test tracks to calibrate parameters in model.

Gazis et al. (1959) developed a non-linear car-following model with a sensitivity term that was inversely proportional to the relative spacing. Subsequently, Edie (1961) assumed that as the speed of traffic stream increases, the driver of the following vehicles will be more sensitive to the relative velocity between the lead and following vehicles. Hence he introduced the speed of the following vehicles into the sensitivity term of the non-linear model developed by Gazis et al. (1959). Concluding this Chapter, we discover that the models used above do not particularly address lane changing manoeuvres in unsignalised intersections which is our main focus for this study. The models already developed use controls such as traffic signals in addressing the traffic congestion challenges in areas where they are employed. Automation of these models also makes it difficult for application on areas where technology is not as advanced. The model adopted in this study is an extension of the Borsche et al. (2012) model where the addition of a lane changing strategy is introduced. The advantage of this model is its ability to take care of the most critical factors in the study of Traffic Congestion such as the minimum acceptable gap, prioritization and lane changing without automation. This effectively means that the model can be employed in non-automized and automized scenarios.

CHAPTER 3

METHODOLOGY

A Microscopic Traffic flow model describes the motion of each vehicle by modeling the action of the vehicles such as the acceleration and deceleration of each driver. This is done as a response to the surrounding traffic by the use of the three strategies; an acceleration policy toward a needed velocity in the free-flow area, a breaking strategy when nearing other vehicles or obstacles, and a safe-car driving strategy for maintaining an appropriately safe distance when driving behind another vehicle.

A microscopic model of traffic flow attempts to analyze the flow of traffic by modeling driver-driver and driver-road interactions within a traffic stream which in that order analyses the interaction between a driver and another driver on the road and of a single driver on the different features of a road. The model usually assumes that drivers react to the inducement from the neighboring vehicles with the main influence coming from the directly leading vehicles. This is known as follow-the-leader or car-following approximation.

3.1 The General motor's car following model

3.1.1 Basic Philosophy

Newtonian mechanics forms the basic philosophy of car-following model; where acceleration may be regarded as the response of a subject to the stimulus, it receives in the form of the force it receives from the interaction with other particles in the system. This basic philosophy of car-following theories can be summarized as follows;

$[Response]_i \alpha [Stimulus]_i$ for the ith vehicle ($i = 1, 2, ...$).

Each driver in the traffic stream can respond to the surrounding traffic conditions by either accelerating or decelerating the vehicle. As seen, different theories on car following have arisen because of the varying views regarding the nature of the stimulus. The stimulus may be composed of the speed of the vehicle, relative speeds, distance headway, etc. It is from this realization that this model advances that General Motors Model by Borsche et al. (2012) to incorporate lane changes and merging abilities.

### 3.1.2 Follow-the-leader model

The car following model proposed by General motors is based on follow-the-leader concept.

This is based on two assumptions;

- The more a vehicle moves at a high speed, the higher will be the spacing between the two following vehicles and

- To avoid collision, a driver must maintain a safe distance with the vehicle ahead

Figure 3.1: Notation for car-following model

17
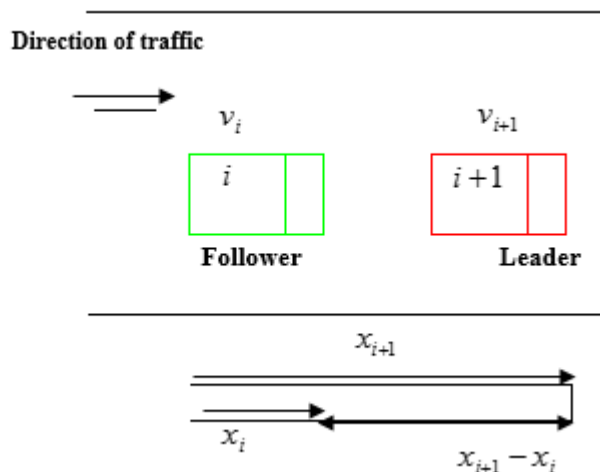
The longitudinal spacing of vehicles is of importance from the points of view of safety, capacity and level of service. This longitudinal space occupied by a vehicle depends on the physical dimensions of the vehicles as well as the gaps between vehicles. To measure this longitudinal space, two microscopic measures used are distance headway and distance gap. The distance headway is defined as the distance from a selected point (usually front bumper) of the lead vehicle to the corresponding point of the following vehicles. Hence, it includes the length of the lead vehicle and the gap length between the lead and the following vehicles.

3.1.3 Model Derivation

The following vehicle is assumed to accelerate at time $t + \Delta T$ and not at t, where ΔT is the interval of time required for a driver to react to a changing situation. The microscopic car-following model in this study is developed in a classical procedure from the so-called 'General Motors' (GM) type car-following models. The model is modified to incorporate lane changing which forms the basis for a comprehensive description of the traffic flow. The location and Speed of the vehicles at time $t\epsilon R^+$ and the distance between the successive cars are denoted respectively as $l_i = x_{i+1} - x_i$ by $x_i(t)$ and $v_i(t)$ and where $i = 1..............n$
Various models were formulated to represent how a driver reacts to the changes in the relative positions of the vehicle ahead (Gunay, 2007; Mathew, 2014). Mathew (2014) and Gunay (2007) discuss various car following models as follows;

3.1.4 Pipe's model

This model assumes that "A good rule for following another vehicle at a safe distance is to allow yourself at least the length of a car between your vehicle and the vehicle ahead for every ten miles per hour of speed at which you are traveling". According to this model, the minimum safe distance headway increases linearly with speed. At low speeds, the minimum head-ways proposed by the theory are

significantly less than the corresponding field measurement; this is a disadvantage of this model.

3.1.5 Forbes' model

This model considers the reaction time needed for following vehicle to perceive the need to decelerate and apply brakes. Simply, it means that the time gap between the rear of the leader and the front of the follower should always be equal to or greater than the reaction time. Thus, the minimum time headway is equal to the reaction time (minimum time gap) and the time needed for the lead vehicle to traverse a distance equivalent to its length. Just like Pipe's Model, the disadvantage of this model is that there's a wide difference in the minimum distance headway at low and high speeds.

3.1.6 General Motors' model

It is the most popular model of all the car-following theories. This is because of the following reasons:

(i) Its agreement with field data; the simulation models developed based on General motors' car following models show good correlation to the field data.

(ii) Its mathematical relation to macroscopic model.

The motion of an individual vehicle is governed by an equation, which is analogous to the Newton's Laws of motion in the car following models. In Newtonian mechanics, acceleration can be regarded as the response of the particle to the stimulus it receives in the form of force which includes both the external force as well as those arising from the interaction with all other particles in the system. In their book Introduction to Transportation Engineering, Mathew (2014) and Gunay (2007) give the most general form of the General Motor's models as below;

Let Δx_{n+1}^t be the gap available for the $(n+1)^{th}$ vehicle, Δx_{safe} be the safe distance, v_{n+1}^t and v_n^t be the velocities.

Then, the gap required given by;

$$\Delta x_{n+1}^t = \Delta x_{safe} + \tau v_{n+1}^t \tag{3.1}$$

where; τ is a sensitivity coefficient. The above equation can as well be written as;

$$x_n + x_{n+1}^t = \Delta x_{safe} + \tau v_{n+1}^t \tag{3.2}$$

Differentiating the above equation with respect to time;

$$v_n^t - v_{n+1}^t = \tau . a_{n+1}^t \tag{3.3}$$

$$a_{n+1}^t = \frac{1}{\tau} \left[v_n^t - v_{n+1}^t \right] \tag{3.4}$$

where; τ is a parameter that sets the time scale of the model and $\frac{1}{\tau}$ can be considered as a measure of the sensitivity of the driver.

Equation 3.4transforms to the most general basic General Motors car following model of the form;

$$a_{n+1}^t = \left[\frac{\alpha l, m \left(v_{n+1}^t \right)^m}{\left(x_n^t - x_{n+1}^t \right)^l} \right] \left[v_n^t - v_{n+1}^t \right] \tag{3.5}$$

where; a is the acceleration of the vehicle is the velocity of the vehicle,l is a distance headway exponent and can take values from $+4$ to-1, m is a speed exponent and can take values from -2 to $+2$, and α is a sensitivity coefficient. These parameters are to be calibrated using field data.

3.1.7 Optimal velocity model

This model is based on the concept that each driver tries to achieve an optimal velocity based on the distance to the preceding vehicle and the speed difference between the vehicles. This was an alternative possibility explored recently in car-following models. The formulation is based on the assumption that the desired speed $v_{idesired}$ depends on the distance headway of the ith vehicle i.e. $v_{idesired} = v^{opt}$ where v^{opt} is the optimal velocity function which is a function of the instantaneous distance headway l_i. Therefore $\dot{v_i}$ is given by;

$$\dot{v_i} = [1/\tau] \left[v^{opt}(l_i) - v_i \right] \tag{3.6}$$

where $1/\tau$ is called as sensitivity coefficient. In summary, the driving strategy of the ith vehicle is that, it tries to maintain a safe speed which in turn depends on the relative position, rather than relative speed./

In the study, a concentration on developing a mathematical model for describing the flow of traffic at unsignalised intersections and then perform simulations of traffic of those unsignalised intersections; by extending the basic GM car following model to incorporate lane-changing manoeuvres that happens at the intersections. A consideration for the generalized General Motors car following model used by Borsche et al. (2012) and is given by the following equation(s) 3.7 and 3.8

$$\dot{x_i} = v_i \tag{3.7}$$

$$\dot{v_i} = C \left(\frac{v_{i+1} - v_i}{l_i - H} \right) + \frac{1}{T}(U(\rho_i) - v_i) \tag{3.8}$$

where;

$\dot{x_i}$ is the velocity of the ith vehicle

$\dot{v_i}$ is the acceleration of the ith vehicle

l_i is the spacing between the preceding vehicle and the following vehicle

H is the length of the vehicle (considered as a constant here)

$U(\rho_i)$ is the equilibrium velocity

T Reaction Time. It is the time a vehicle takes to adjust to the equilibrium velocity. (The time the ith vehicle takes to move from one point to the next in response to the stimuli induced). Also, ΔT is the interval of time required for a driver to react to a changing situation.

C is a constant. It scales the anticipation term

The first term and second term in the acceleration equation are the anticipation and relaxation terms respectively.

From the above discussions, we now narrow down to the use of the following the microscopic equations;

$$\dot{x_i} = v_i \tag{3.9}$$

$$\dot{v_i} = C\left(\frac{v_{i+1} - v_i}{l_i - H}\right) + \frac{1}{T}(U(\rho_i) - v_i) \tag{3.10}$$

The local "density around vehicle i" and its inverse (the local (normalized) "specific volume") are respectively defined by;

$$\rho_i \frac{H}{l_i}$$

and

$$\tau_i = \frac{1}{\rho_i} = \frac{l_i}{H}.$$

A car-following model describes the motion of the following vehicles in response to the leading vehicle. The main task of the model is to reproduce a car following symbolism in a real-life scenario; it is prudent then to compare this to the driving behaviour of human drivers so as to come up with an empirical result. Basically

all vehicles that are not in free flow, adjust their driving behaviour to the vehicle ahead, to keep a safe headway. In theory, car following models can be used for all kinds of vehicles.

Figure 3.2: Description of Movement along a T-Junction
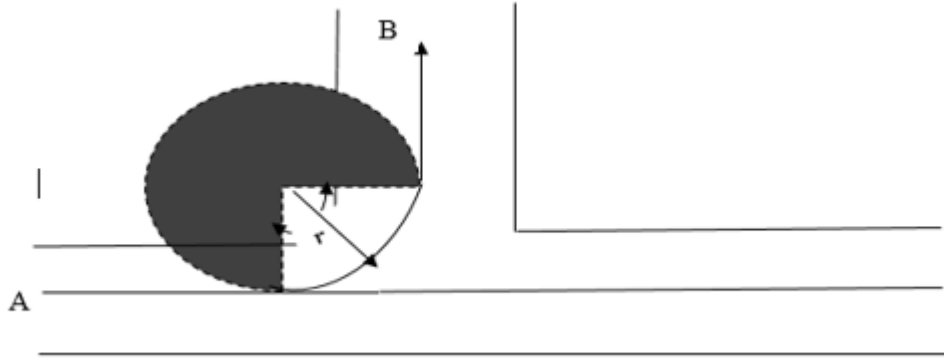
Using Figure 3.2 above, a description on how a vehicle moves along a T-junction is done. This is began by giving the initial conditions as $0 < \theta < 2\pi$. A T-Junction is considered where a vehicle moves from the straight path and to the connecting path. Notably, this motion makes an arc like path. A vehicle (here viewed as particle) moving from point A to B is considered. To better understand and analyze this motion the curve (arc) parameterized.

This is achieved by we invoking the polar coordinates. It is imagined that if this path continued, the vehicle moves in such a way that it forms a circle with a center somewhere and with radius . The movement is occurring on a two-dimensional plane on the x and y axes where considering x and y in terms of the polar coordinates; $x = r cos\theta$ and $y = r sin\theta$. Here, r is constant throughout the motion. Only what changes is θ as the vehicle moves along path AB. Looked this way, r is a vector and is given as $\vec{r} = r(cos\theta_i e_1 + sin\theta_j e_2)$. A velocity v is applied on the vehicle as it travels through the desired distance x.

3.2 Mathematical Formulations

3.2.1 Assumptions of the Model

(i) Priority is given to vehicles on the major road.

(ii) Vehicles respond to frontal stimuli

(iii) Vehicles have the same lengths and have the same acceleration capabilities. This applies to all vehicles

(iv) The intersection considered is in a city and the time for this consideration is during rush hours (Early Morning or in the Evening). This ensures there's enough supply of vehicles at the intersection

(v) The drivers are cooperating (they are sane)

3.2.2 Derivation of the Equations of the model.

Now considering a dynamical system of i vehicles, where $i \epsilon n$, translated as below;

$$x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ . \\ . \\ . \\ x_n(t) \end{pmatrix}$$

The ith component of the $x(t)$ is $x_i(t)$ and $x(t)$ is again the vector position.

By differentiation;

$$v(t,x) = \frac{dx}{dt} = \begin{pmatrix} \frac{d}{dt}x_1(t) \\ \frac{d}{dt}x_2(t) \\ . \\ . \\ . \\ \frac{d}{dt}x_n(t) \end{pmatrix}$$

In a general setting, it can be seen that $\frac{dx}{dt}f(x,t)$ where f stands for the vector field.

In the setting of this introduction, the aim is to propose the studied model with additional details in how to approximate the trajectory, in particular those translating the motion at the junction.

By integration $\frac{dx}{dt} = v(t,x)$ with respect to time;

$$\int \frac{dx}{dt}dt = \int v(t,x)dt$$

From the equality below, an approximation for the curved line measuring the turning angle θ, where θ depends on t is introduced.
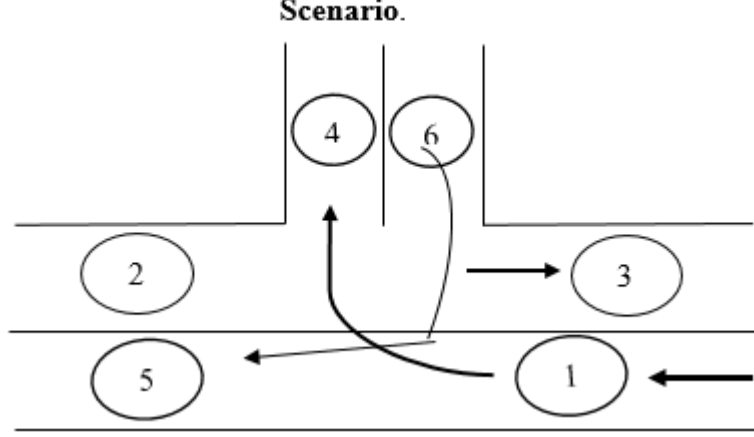
$$x(t) = \int v(t,x)dt$$

For each vehicle i, its position $x_i(t)$ is given by;

$x_i(t) = \int v_i(t,x)dt$; where $i = 1, 2, ..., n$

Studying the scenario presented by Figure 3.3 below, it is noticed that there'll be two types of movements;

- Straight Movements e.g. $1 \to 5$, $2 \to 3$.

- Turning Movements e.g. $1 \to 4, 6 \to 5$

Figure 3.3: Interactions of vehicles at a T-Junction

Scenario.

Let l be the distance between the vehicles; Let x_1 be the vehicle at position 1 intending to go to position 4 and Let x_2 be the vehicle at position 2 moving straight to position 3

Now, at time instance t^*, vehicle v_1 can only get to 4 *iff* there's a safe distance between itself and vehicle x_2, that is;

$$x_1 - x_2 =\geq l_{safe}(x_1, x_2, v_2)$$

Similarly, for vehicles at position 6 going to 5, applying the same conditions as described above; it has to check vehicles x_1 and x_2 this motion is described as below;

$x_6 - x_2 =\geq l_{safe}(x_6, x_2, v_2)$ and $x_6 - x_1 =\geq l_{safe}(x_6, x_1, v_1)$.

Studying the velocities of the two movements its is found that the straight movements velocities is described by 3.9.

While for the Turning movements the vehicle velocity is described by;

$$\dot{x_i}(t) = r\left[cos\theta_i(t)e_1 + sin\theta_j(t)e_2\right] \tag{3.11}$$

where;

$$\theta_i(t) \cong \int v_i(t)dt, \tag{3.12}$$

and e_1 and e_2 are unit orthogonal vectors and θ describes a movement along a curve which translates to a trajectory which is measured a change in θalong the x and y axes.

In both cases, the acceleration for the ith vehicle is described by 3.7 where the constant $C > 0$, and reaction time T are the given parameters. The function $U = U(\rho), 0 \le \rho \le \rho_m$ is the so called fundamental diagram. The simplest choice is given by $U(\rho) = 1 - \rho$.

Combination of both the velocity and acceleration for the straight movement of the vehicles yields to the following Microscopic Equations 3.6and 3.13.

$$\dot{v}_i = C\left[\frac{v(t)_{i+1} - v(t)_i}{\tau_i - H}\right] + \frac{1}{T}(U(\rho_i) - v(t)_i) \tag{3.13}$$

where τ is a parameter that sets the time scale while $\frac{1}{T}$ is a measure of the sensitivity of the driver as he/she moves to attain the equilibrium velocity.

From $\rho_i = \frac{H}{l_i}$ and $\tau_i = \frac{1}{\rho_i} = \frac{l_i}{H}$, and with reference to the acceleration equation, this transformation is obtained;

$\dot{l}_i = v_{i+1} - v_i$ or $\dot{\tau}_i = \frac{1}{H}(v_{i+1} - v_i)$

From the above movement description, the model equations for both the straight and turning movements -for both velocities and acceleration- are obtained and represented as below;

- **Straight Movement:**

$$\dot{x}(t) = v(t) \tag{3.14}$$

$$v\dot{results}(t) = C\left[\frac{v(t)_{i+1} - v(t)_i}{l_i - H}\right] + \frac{1}{T}(U(\rho_i) - v(t)_i) \tag{3.15}$$

27

- **Turning Movement**

  Equations 3.11 and 3.15.

3.2.2.1 Discretizing using the Forward Finite Difference method.

In microscopic traffic modeling, each vehicle is considered on its own. Therefore, a discretization is done to get the equations representative of each vehicle. This is done by the use of the finite forward finite difference method.

$$\frac{dx_i}{dt} = v_i$$

$$\dot{x} = v$$

Now, letting k be the time index and i the location index;

$$\frac{dx_i}{dt} = \frac{x_{i+1}^{k+1} - x_i^k}{\triangle t}$$

$$\frac{x_{i+1}^{k+1} - x_i^k}{\triangle t} = v_i$$

where x_{i+1}^{k+1} is the position of the front bumper of the leading vehicle and x_i^k is the position of the front bumper of the following vehicle

Starting with n as zero;

$$\frac{x_i^1 - x_i^0}{\triangle t} = v_i$$

For the acceleration function;

Let k be the time index;

Then;

$$\frac{dv_i^k}{\triangle t} = \frac{v_{i+1}^{k+1} - v_i^k}{\triangle t}$$

where; v_{i+1}^{k+1} is the velocity of the leading vehicle and v_i^k is the velocity of the following vehicle.

Then;

$$\frac{v_{i+1}^{k+1} - v_i^k}{\triangle t} = C \left[\frac{v_{i+1} - v_i}{l_i - H} \right] + \frac{1}{T}(U(\rho_i) - v_i)$$

Since the constants C and T are known, it'll be easy to substitute them in the equation hence get the acceleration of the vehicle under consideration. This equation is used for each individual vehicle.

3.2.3 Existence and Uniqueness of Results

It is very important for us to check if the results exist and if yes, if they are unique. This is done by invoking the Cauchy Lipschitz Theorem.

Statement of the Cauchy Lipschitz Theorem

Let $U \subset R^n$ be an open set and $f : U \times [0, T] \to R^n$ a continuous function which satisfies the Lipschitz condition $|f(x_1, t) - f(x_2, t)| \leq M |x_1 - x_2| \, \forall (x_1, t), (x_2, t) \epsilon U \times [0, T]$ (where M is a given constant). Let us consider the initial value problem described by $\begin{cases} \dot{x} = f(x, t) \\ x(0) = x_0 \end{cases}$ where $x \epsilon U$ and $t \epsilon [0, T]$. If $x_0 \epsilon U$ then for some positive δ, there is a unique solution $x : [0, \delta] \to U$ of the initial value problem.

Remarks:

(i) A similar statement holds if $[0, T]$ is replaced by $[-T, 0]$ or $[-T, T]$ (the interval of existence becomes then, respectively, $[-\delta, 0]$ and $[-\delta, \delta]$).

(ii) The existence is limited to a small interval because the integral curve x might "leave" the domain U. In particular, either δ can be taken equal to T or there is a maximal time interval of existence $[0, T_0[$ of the solution, characterized by the property that $x(t)$ approaches the boundary of U as $t \to T_0$.

By considering an hypothesis in the model where the lipschitz condition is satisfied and then subjecting that to the model equations 3.14,3.11 and 3.15 where: e_1 and e_2 are unit orthogonal vectors

$\dot{x_i}$ is the velocity of the ith vehicle

$\dot{v_i}$ is the acceleration of the ith vehicle

l_i is the spacing the preceding vehicle and the following vehicle

H is the length of the vehicle (taken as a constant in this study)

$U(\rho_i)$ is the equilibrium velocity and

T is the Reaction Time. I.e. the time a vehicle takes to reach the equilibrium velocity. Also, $\triangle T$ is the interval of time required for a driver to react to a changing situation

C is a constant which scales the anticipation term

The first term and second term in the acceleration equation is the anticipation and relaxation terms respectively.

Since the equations represent vehicles and by considering n vehicles from the above sets of ODEs then;

$$Z = F(Z)$$

where,

$$Z = \begin{pmatrix} \dot{x_i} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \dot{x_n} \\ \dot{v_i} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \dot{v_n} \end{pmatrix}$$

and also

$$F(Z) = \begin{pmatrix} v_1 \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ v_n \\ C(\frac{v_2(t)-v_1(t)}{l_i-H}) + \frac{I}{T}(U(\rho_i) - v_1) \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ C(\frac{v_{n+1}(t)-v_n(t)}{l_n-H}) + \frac{I}{T}(U(\rho_n) - v_n) \end{pmatrix}$$

$$Z = G(Z)$$

where Z is the same as above and

$$G(Z) = \begin{pmatrix} rcos\theta_1 e_1 + sin\theta_1 e_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ rcos\theta_n e_1 + rsin\theta_n e_2 \\ C(\frac{v_2(t)-v_1(t)}{l_i-H}) + \frac{I}{T}(U(\rho_i) - v_1) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ C(\frac{v_{n+1}(t)-v_n(t)}{l_n-H}) + \frac{I}{T}(U(\rho_n) - v_n) \end{pmatrix}$$

Taking the hypotheses as in the Cauchy Lipschitz theorem, $\dot{x}(t) = v(t)$ and $\dot{x}(t)$ is a continuous vector function and also $\dot{v}(t)$ is a Lipschitz continuous vector function. Then $x(t)$ and $v(t)$ are continuously differentiable. The solution of each system of ordinary differential equation is called the integral curve.

Drivers are governed by the following set of rules;

(i) In same speed (synchronized) flow, drivers accept a range of different hypo-thetical steady states in the flow regime.

(ii) To avoid collisions in the steady states, a driver does not accept a safety distance which is higher than the one specified.

Equipped with this knowledge, the simulation of vehicles is done on this T-Junction intersection. Velocities are always positive and keep updating themselves as the vehicles are within the area of consideration of in the T-Junction. In the simulation, vehicle dynamics are less important than driver interactions in simulating queue formation in the unsignalised intersections. The timings for all vehicles under consideration are counted in seconds at all times in the whole study.

Further, the simulation takes care of driver interactions. The model attempts to analyze the flow of traffic by modeling driver-driver and driver-road interactions within a traffic stream, which respectively analyses the interaction between a driver and another driver on the road and of a single driver on the different features of a road. Theoretically, a vehicle at the stop line of a minor stream can drive onto the intersection, without interrupting the primary flow, when the space between two vehicles on the major stream is big enough.

Remark. *Mathematical modeling has shown how a combination of congestion and the behaviour of an individual driver can artificially create a traffic jam. If traffic is flowing smoothly on a roadway with a high volume of drivers and a single distracted driver aggressively brakes to slow down, then sudden braking can propagate backwards as a wave, bringing traffic to a standstill on the roadway several miles behind. In this study, a decision on the minimum acceptable gap that a vehicle can keep among itself, the leading and the following vehicles.*

Again a consideration about the stop and go conditions in the simulation process is done. According to the rules of the road, a vehicle from a minor-street has to obey a stop sign before it can enter an intersection. The simulation ensures that all vehicles from the all lanes stop and then go depending on the priorities of the lanes they are on.

The simulation of the car following model is done using the C++ programming language.

Note: The intersection chosen is in a place where the rules of priority are existent and where all the main roads and minor streams/roads have non-equal ranks in the hierarchy of departure mechanism.

3.3 The Simulation Process

3.3.1 Real World to Virtual World Representation

The Real world is complex, full of diverse objects and activities that are unique in their own ways. A single instance of the real world contains a subset of these billions of objects and activities. The computer world is limited in terms of memory, computation power and the fact that it can never be said to be 100% accurate. Therefore, to solve a real world problem computationally, various issues are considered. These may include transformations, approximations, assumptions, isolation etc. That's computational world is just a subset of a particular world instance. Thus, in the actual computed solution of a real world problem, only particular important objects and activities of the real world subject to the anticipated solution are put into consideration.

A road is one of the objects of the real world. This object is involved in various activities which are described by its interaction with other real world objects such as humans, cars, animals, other roads, etc. Any computer program aiming at solving a road-related problem must vividly define how these various objects

and activities will be represented in the computer world. Such a solution may or may not consider the whole sets of objects and activities associated with a road. It's purely dependent on the problem at hand.

In this study, the major goal is to find out how traffic jams develop at unsignalised intersections. This defines the objects of interest to be a junction, vehicles and drivers, and activities to be considered are movement of the vehicles on the road, driver's decisions regarding the movement of the vehicles and the turning of the vehicles from one lane to another on the road. Therefore, we think on how these objects and activities are represented in the computer world.

There are various ways in which a road may be represented in the computer. Only road's dimension will be mapped to the computer world. Again, vehicles' movement to be considered is a certain amount of distance away from the junction, which was decided to be 200m on all the three sides of the T-junction. 200m was voted in because it was found to be sufficient for the research at hand. With this known, it was found to be more efficient to map the road's dimension to Cartesian plane. This would provide easier platform for mapping other objects and activities such as vehicles and movements respectively.

A T-junction comprises of a major road which runs past the junction with a minor road as shown in the figure below.
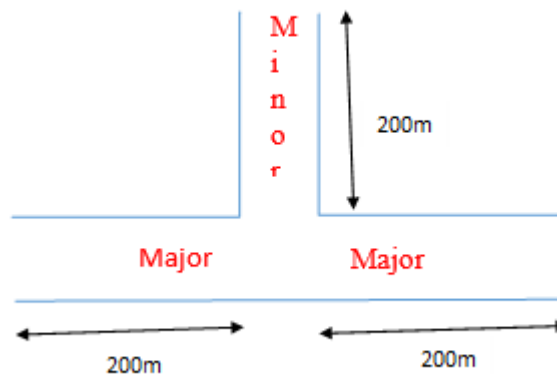
Figure 3.4: T-Junction with Major and Minor Roads

The goal was to map such a road to a Cartesian plane in a computer world. Having no computer of such large screen, we needed to do some approximation of the road. From mathematics, this is achieved by use of a scaling factor. A computer screen is measured in pixels. Therefore, a screen of X-by-Y pixels was used to draw a Cartesian plane of R-by-Q units where one pixel was used to represent some $n - units$.

Therefore, the most important question was how many meters does a single Cartesian unit represent. A scale of one unit represents 10 meters was found to be sufficient. This implied a Cartesian of 40 $X - units$ and 20 $Y - units$ suffices. To represent other important data such as the road width as well as appealing presentation of the road to the user, a Cartesian plane of 43 $X - units$ and 25 $Y - units$ was chosen. Where the $43 units$ were taken from 0 to 43 and the $25 units$ were taken from $-2\ to\ 23$ as shown in the figure below.

Figure 3.5:   Cartesian Plane Representation of the Road
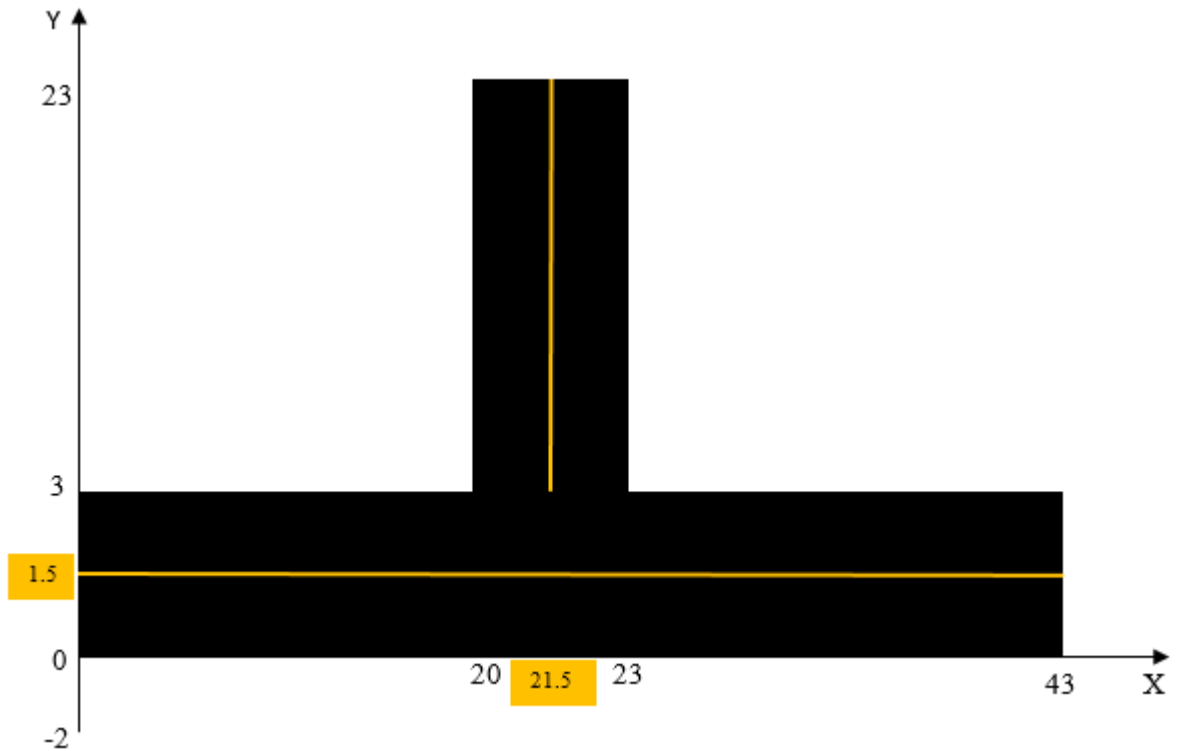
The other object that needed to be represented is a vehicle that moves on a

37

road. The vehicle can be mapped to various shapes including rectangle, circle, a tree or anything so long it is clearly defined what such a shape on the Cartesian plane does represent in the real world. It was found to be wise to implement the vehicles as circles of 1 *units*. This allows flexibility especially in mapping the movement and turning activities of the vehicles. That is, turning a circle is a better representation of the turning activity in the real world than when turning a rectangle.

The activities in the real world must also be mapped to some given activities in the computer world. The movement of the vehicles was mapped to moving the circles described above along either $X - axis$ from point $(0, 0)$ to point $(43, 0)$ or $Y - axis$ from point $(0, 0)$ to point $(23, 0)$ and vise versa. Turning activity was mapped to changing vehicles movement from moving along $X - axis$ to moving along $Y - axis$ and vise versa. Drivers' decisions such as stopping of the vehicles and acceleration were mapped to stopping a given circle at a given point for as long period as the driver stopped and moving vehicle for varying number of units per unit time respectively.

The turning activity was quite complex one that involves changing from one lane to another lane perpendicular to the previous lane. That is, the vehicles movement is shifted to moving along a path that is 90^0 away. In real world, this activity would just involve turning the steering either clockwise or anticlockwise direction until the vehicle fully negotiates through the 90^0 angle. In the computer world, nothing like a steering does exist. Thus, one has to find a way of changing the angle of the path that a vehicle is moving along to a path located at the 90^0 turn.

It is known that *cosine* and *sine* of any angle lies between -1 and 1 inclusive. It is also known that having a positive integer N, the quotient, n/N, lies between 0 and 1 inclusive.
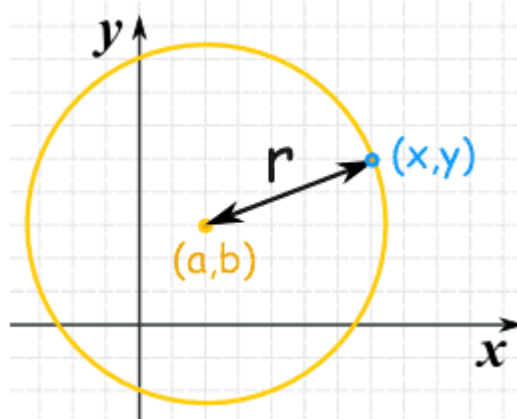
Thus, if you add an auxiliary equation, $n/ - N$, where $n = 0, 1, 2, ..., N$, gives a

quotient between -1 and 0 inclusive.

Therefore, the two equations combined together will give quotient between -1 and 1 inclusive.

It is also known that a circle is a locus of points on a plane that are a fixed distance, radius r from a given point, center of the circle (a, b).

Figure 3.6:   Locus of Points of a Circle

On a Cartesian plane, these points are computed as follows. Lets make a right-angled triangle as shown below;

Figure 3.7:   Computation of points of a circle
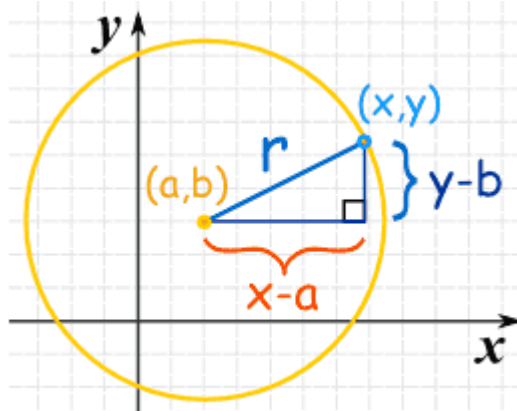
By the Pythagoras theorem,

$$c^2 = a^2 + b^2$$

$$r^2 = (x - a)^2 + (y - b)^2$$

Clearly it can be seen that for us to draw the circle when the radius r and the center (a, b) are known, we need to vary the values of x and y. Remember these are located at the circumference of the circle to be drawn. We therefore modify the circumference formula to get the x and y values along the circumference as described.

$Circumference, C = 2\pi r$

The circumference of a curve defined by angle θ is compute by,

$Circumference, C = r * 2 * \theta / 2\pi$

Assuming θ is unknown, we compute it as follows;

Let N be a positive integer. Then,

$\theta^0 = 2\pi * n/N$, where, $n = 0, 1, 2, 3, ..., N$ and $\theta^0 \leq \theta \leq 2\pi$

But it is known that when radius r and center (a, b) of the circle are known and given angle θ^0, then point (x, y) at θ^0 along the circumference can be computed as follows;

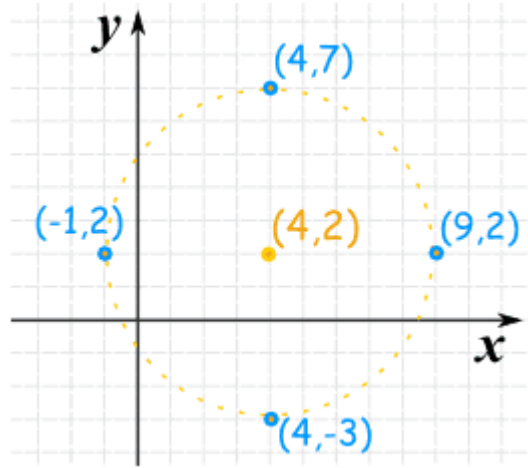$x = a + r * cos(\theta^0)$

$y = b + r * sin(\theta^0)$

But,

$0 = 2\pi * n/N$, thus, when n is known, the point (x, y) is given by;

$x = a + r * cos(2\pi * n/N)$,

$y = b + r * sin(2\pi * n/N)$

Therefore, to draw the we needed to vary the n from 0 to N. These points defined the points on the circumference of the circle. Thus, we got a full circle by connecting these points using a smooth curve. See the example below;

Figure 3.8:   Circumference Representation of a circle

With a graphics software, these points were passed to it, which in return did the drawing job. OpenGL is one such kind of a library that provides various commands for drawing primitives such as lines, triangles, line strips, etc. To draw a complex figure, one has to decompose it into its various primitives. A circle is a strip of isosceles triangles as shown below.

Figure 3.9:   A Circle as a strip of isosceles triangles
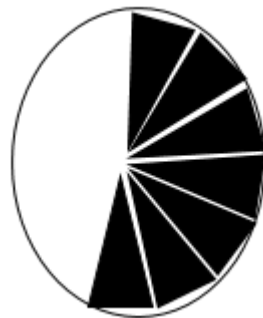
If the base of the triangles is made so small such that the triangle resembles a line, the circumference of the circle becomes so smooth that the naked eye can't notice the stripping of the triangles. We therefore used OpenGL to draw such triangles in a closed loop. See the OpenGL C++ code snippet below.

$glBegin(GL\_TRIANGLE\_FAN)$;

$glVertex2f(x, y);$

$for(int\ i = 0; i <= carTriangleAmount; i++)\{$

$glVertex2f(x + (carRadius * cos(i * twicePi/carTriangleAmount)),$

$y + (carRadius * sin(i * twicePi/carTriangleAmount)));$

$\}$

$glEnd();$

Where $x = a$, $y = b$, $carRadius = r$, $i = n$, $twicePi = 2\pi$, $carTriangleAmount = N$.

The above code draws the circular vehicles.

Turning a vehicle involves moving the vehicle along the circumference of a given quadrant in the clockwise or ant-clockwise direction. Each of the four quadrants is defined by an angle of 90^0. We therefore computed the points along such a quadrant using the formula discussed earlier and the facts we know about the various quadrants. Take for example, we know that the third quadrant is defined by $270^0 \leq \theta^0 \leq 360^0$. That is, $3/4N \leq n \leq N$. Thus, to turn a given vehicle along the third quadrant of a circle with some known center and radius in ant-clockwise direction, we needed to compute the various points lying along the quadrant and use these points as the centers of the circular vehicles. To turn it in the clockwise direction, we just change N to $-N$.

In the simulation radii 0.7 and 2.3 were used to turn the vehicles along left and right turns respectively. Remember the road is 20 units long from the junction, and each road is 3 units wide. Therefore, from point $(0, 0)$ to the junction, there are 20 units which a vehicle has to cover before it enters the junction. Each road is divided into two equal halves to represent the left and right sides of the road. To do this, a yellow line is drawn along the $x - axis$ through.

$y = (3 - 0)/2 = 1.5$, *"Remember the main road width starts at $y = 0$ to $y = 3$"*

Likewise, a line is drawn along the y-axis through,

$x = 20 + (3.2) = 21.5$, *"Remember the minor road width starts at $x = 20$ to*

$x = 23$"

Vehicles on a Cartesian plane move along a given axis where a certain coordinate is kept constant either for x or y. In this plane for this study, vehicles move along the $x - axis$ via lines $y = 0.7$ and $y = 2.3$, and along the $y - axis$ via lines $x = 20.7$ and $x = 22.3$. These values were obtained by trial-and-error method. But remember that the vehicles of size one unit, therefore the radius is 0.5 $units$. Assume the vehicle is moving along $x - axis$ via $y = 2.3$, then there is a mall gap between the vehicle and the left side of the road. This gap is given by $(3 - (2.3 + 0.5)) = 0.2$ $units$. This is because 2.3 is the y value of the center of the circle representing the vehicle. Clearly, it can be seen that the distance between the center is given by $(gap + radius = 0.2 + 0.5 = 0.7$ $units)$. This value defines the turning radius of the vehicle from the main road to the minor road.

Figure 3.10: Movement of vehicles on the Cartesian Plane
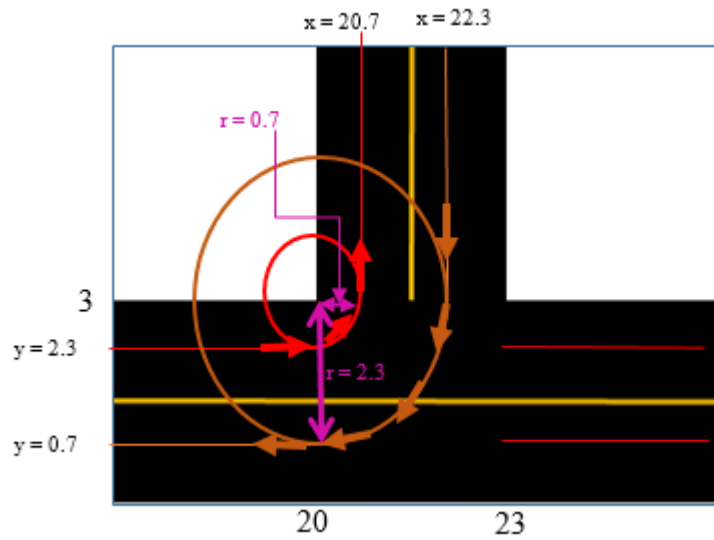
The same applies for the right turning from minor road to the main road. Since the vehicle turns from $x = 22.3$ to $y = 0.7$, the center of the quadrant along which the vehicle turns is still $(x, y) = (20, 3)$. The radius is given by $(3 - 0.7 = 2.3)$. If the vehicle wants to turn left from main to minor road, it moves along $x - axis$ on $y = 2.3$ until it reaches at $x = 20$. It then turns along the circumference of

43

the fourth quadrant of the circle centered at $(x, y) = 20, 3)$ in the ant-clockwise direction until it reaches the point where $x = 20.7$ and proceeds along the $y-axis$. See the code snippet below;

$x\_centre = 20.0$;

$y\_centre = 3.0$;

$//leftTurnRadius = 0.7$

$//carTurnAngleModifier = 16$

$//leftTurnTriangleAmount = 20$

$x = x\_centre+(leftTurnRadius*cos(carTurnAngleModifier*twicePi/leftTurnTriangleAm$

$y = y\_centre+(leftTurnRadius*sin(carTurnAngleModifier*twicePi/leftTurnTriangleAm$

$carTurnAngleModifier + +$;

In a similar manner, if the vehicle wants to turn right from minor to main road, it moves along $y-axis$ on $x = 22.3$ until it reaches at $y = 3$. It then turns along the circumference of the fourth quadrant of the circle centered at $(x, y) = 20, 3)$ in the clockwise direction until it reaches the point where $y = 0.7$ and proceeds along the $x-axis$.

$x\_centre = 20.0$;

$y\_centre = 3.0$;

$//rightTurnRadius = 2.3$

$//carTurnAngleModifier = -59$

$/rightTurnTriangleAmount = -60$

$x = x\_centre+(rightTurnRadius*cos(carTurnAngleModifier*twicePi/rightTurnTriangle.$

$y = y\_centre+(rightTurnRadius*sin(carTurnAngleModifier*twicePi/rightTurnTriangle.$

$carTurnAngleModifier + +$;

The turning logic purely utilizes the equations of a circle to turn the circular vehicles along various quadrants of interest. We have also seen that Cartesian plane has been used as the major platform for mapping real world objects to computer world objects.

The model equations 3.11, 3.12, 3.13, 3.14 and 3.15 form the core part in the simulation model. In the computer, implementation of the simulation model, three things need to be remembered:

(i) A driver will react to the change in speed of the front vehicle after a time gap called the reaction time during which the follower perceives the change in speed and reacts to it.

(ii) The vehicle position, speed and acceleration will be updated at certain time intervals depending on the accuracy required. Lower the time interval, higher the accuracy.

(iii) Vehicle position, speed and the acceleration are governed by the car following model.

The following diagram represents the T-Junction under consideration

Figure 3.11: The Simulated T-Junction
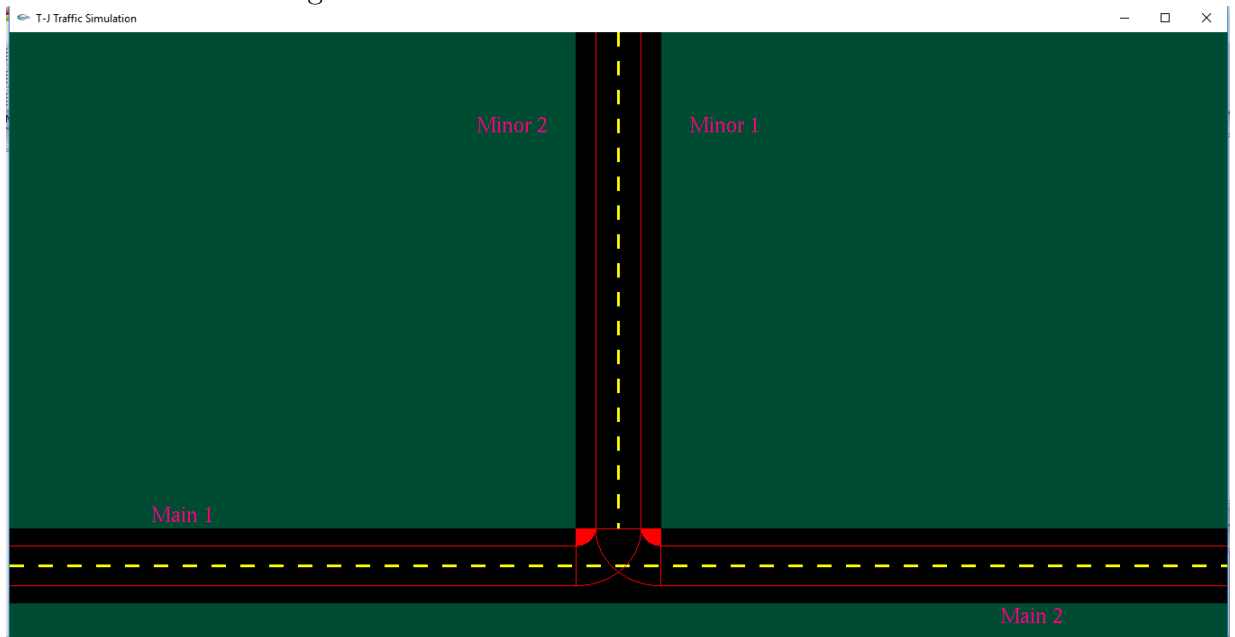
3.3.2 Car Generation

There is a method called car generator that is used for the generation of cars after some specified time interval. The smaller the time interval, the higher the number of cars generated per second. Once a vehicle is generated the method randomly assigns a lane and turn to this vehicle. The method is also responsible for randomly determining the vehicle's color and all the other vehicle's initial properties such as velocity and the appropriate position used to present the vehicles on the Cartesian plane as is in the real world.

3.3.3 Lane Assignment.

There are three lanes. Simple probabilities have been used to assign vehicles to lanes where $Main2$ has the probability of 0.5, Main1= 0.35 and Minor1= 0.15.

3.3.4 Turning Determination.

For each lane, there are only 2 possible movements that a vehicle can make at the junction i.e. vehicles in $Main1$ may turn left to $Minor2$ or go straight, vehicles in $Main2$ can turn left to Min$or2$ or go straight and $Minor1$ may turn left to $Main1$ or turn right to $Main2$. Therefore, each option is given a probability of 0.5.

3.3.5 The Driving Logic

Assuming that a vehicle was assigned some velocity v at the beginning by the car generation function, the velocity and the position are updated using the general motors car following model as follows;

Position Updating; in simulation, parameters are updated after a time interval dt. After updating time interval has elapsed (dt), two conditions must be ascertained to be true;

(i) The vehicle is in-front, or

(ii) The gap between the following and the preceding vehicle is greater the safe distance.

Assuming one of the above conditions is true, then the position is updated by $dt \times velocity$.

Velocity Updating: velocity of a vehicle is also updated after interval dt where 2 conditions are considered;

(i) If the vehicle is in-front, then, velocity is not updated.

(ii) Else it is updated using the car following model.

The vehicles' position and velocity are updated using the GM car following model iff the car hasn't turned or past the intersection (junction). If the vehicle is past the junction, it is assumed that its behaviour doesn't affect the vehicles at or before the junction, therefore, its position is updated uniformly by adding a predefined constant value irrespective of the velocity previously held.

3.3.5.1 The Turning Logic:

At a T-Junction, there are six possible movements that a vehicle can take. Different turns if taken at the same time will result to collisions. To avoid collisions in real life scenarios, different movements have different priorities. Consequently, to avoid collisions in this simulation, similar rules are applied as described here below;

(i) If a vehicle wants to turn from Main1 to $Minor2$, it MUST ensure no vehicle is turning form $Main2$ to $Minor2$.

(ii) If the vehicle is going straight past the junction in $Main1$, it MUST ensure condition 1 above is satisfied and no vehicle is turning at all from $Minor1$ else it should wait.

(iii) If the vehicle wants to go straight past the junction in $Main2$, it MUST ensure no vehicle is turning right from Minor1.

(iv) If the vehicle wants to turn right from $Main2$ to $Minor2$, it MUST ensure, no vehicle moving straight in $Main1$ and no vehicle turning right from $Mionr1$ to $Main2$

(v) If the vehicle wants to turn right from $Minor1$ to $Main2$, it MUST ensure no vehicle in $Main2$ is either going straight or turning right and no vehicle in $Main1$ is going straight.

(vi) If the vehicle wants to move from $Minor1$ to $Main1$, it MUST ensure no vehicle is moving straight on $Main1$.

To achieve the prioritization witnessed in the real world scenario, the simulation begins by processing vehicles in $Main2$ followed by vehicles on $Main1$ and finally $Minor1$ because single vehicle is processed after at a time in the simulation cycle.

3.3.5.2   Turn Angle Modification:

It is known that *sine* and *cos* vary between -1 and 1 for both positive and negative angles. A vehicle turning left or right will move along a quadrant circumference governed by angle variation determined as follows;

(i) When the vehicle turns left from $Main1$ no $Minor2$, it moves along the circumference of the fourth quadrant in the positive direction (anti-clockwise direction), i.e. $270 < \theta < 2\pi$.

(ii) When the vehicle turns left form $Minor1$ to $Main1$, it moves along the circumference of the third quadrant in the positive direction (anti-clockwise direction), i.e. $\pi < \theta < 270$.

(iii) When the vehicle turns right form $Main2$ to $Minor1$, it moves along the circumference of the third quadrant in the negative direction (clockwise direction), i.e. $270 < \theta < \pi$.

(iv) When the vehicle turns right form $Minor1$ to $Main2$, it moves along the circumference of the fourth quadrant in the negative direction (clockwise direction), i.e. $2\pi < \theta < 270$.

3.3.5.3 Stop and Go Conditions:

A driver will always try to maintain a safe distance between the preceding and the following vehicle therefore whenever the gap between the two vehicles is less than some safe distance the driver will tend to slow down or stop. In this simulation, the vehicles attempt to maintain a gap of 3 meters. The vehicles may also stop to give way to a turning vehicle. In such a situation, the stopping effect of the leading vehicle will be propagated backwards to the following vehicles in the same manner as mentioned in the safe distance explanation.

CHAPTER 4

RESULTS AND DISCUSSIONS

The results in the form of Snapshots and Graphs for selected traffic scenarios are presented here. Also, the generated car log and the generated velocities for the different cars is also presented.

In the graphs, the Red Curve represents the leading vehicle, the Blue Curve represents the following vehicle while the Green Curve represents the turning vehicle.

Assumption: *All vehicles begin with the same velocity.*

4.1 Scenario 1: Position-Time Snapshot and Graph for First 3 vehicles in the Simulation

Figure 4.1: Position Time Snapshot for First 3 vehicles in the Simulation
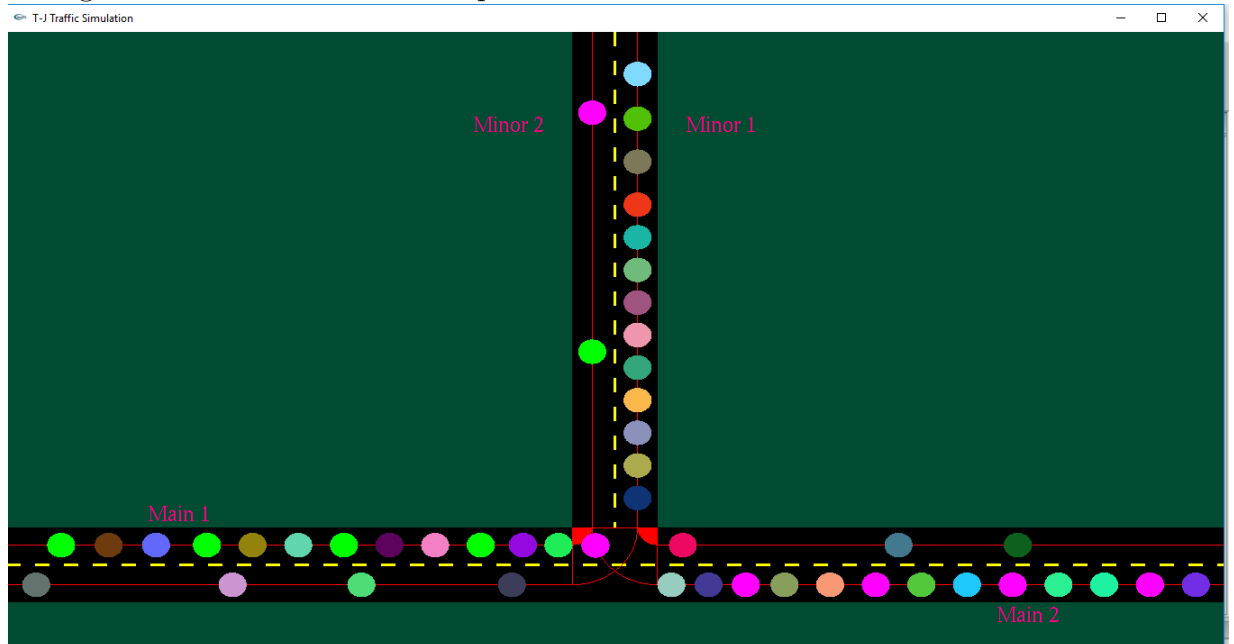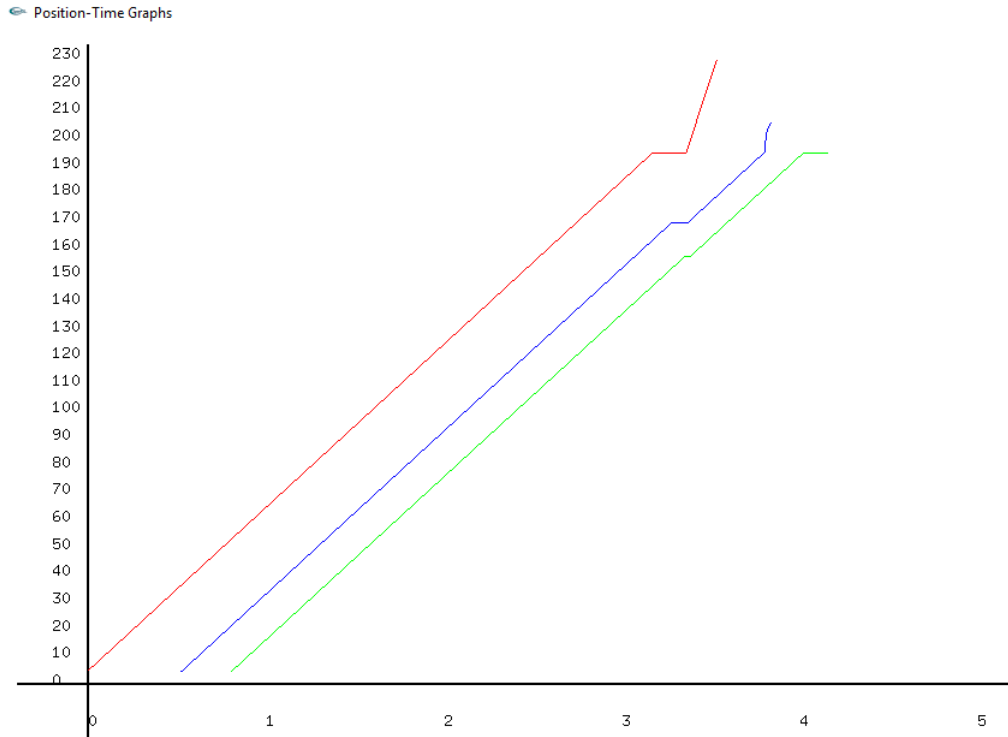
Figure 4.2:   Position Time Graph for First 3 vehicles in the Simulation

Here above, the leading car changes position uniformly with respect to time and its unchanging velocity. When it reaches the junction, the vehicle momentarily waits for another vehicle to turn after which it proceeds to drive past the junction at a higher speed hence the sharp gradient and the vehicle exists the junction. The following vehicle (blue vehicle) accelerates using the General Motors Car following model equations therefore the vehicle solely dependent on the preceding vehicle (red). This is seen at the point where the following vehicle takes the behaviour by stopping and moving when it (leading) starts moving respectively. The Turning vehicle's (third and green) behaviour is as well depicted by the characteristics of the behaviour of the second vehicle. As seen in the Graph, the vehicle stops for some time and resumes motions as soon as the leading vehicle (Second) resumes movement. This clearly shows that the behaviour of the first vehicle was back-chained to all the following vehicles via the follow the leader behaviour as described by the General Motors Car Following Model.

4.2 Scenario 2: Position-Time Snapshot and Graph for the 11th, 12th and 13th Vehicles in the Simulation

Figure 4.3: Position Time Snapshot for the 11th, 12th and 13th Vehicles in the Simulation
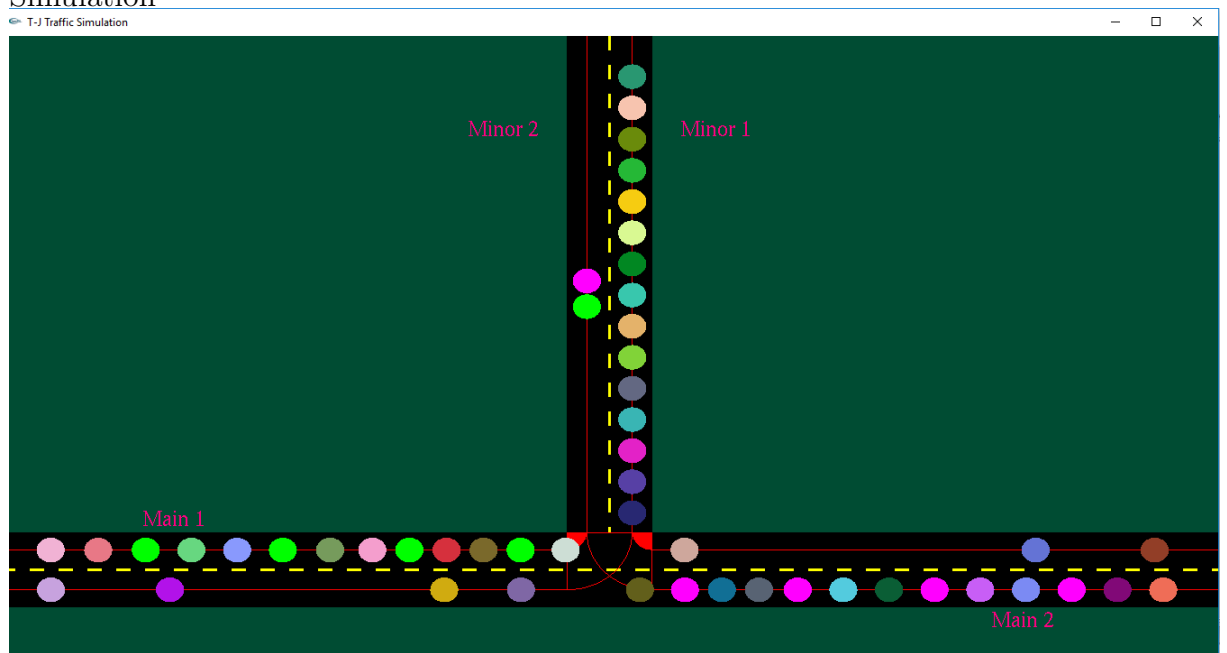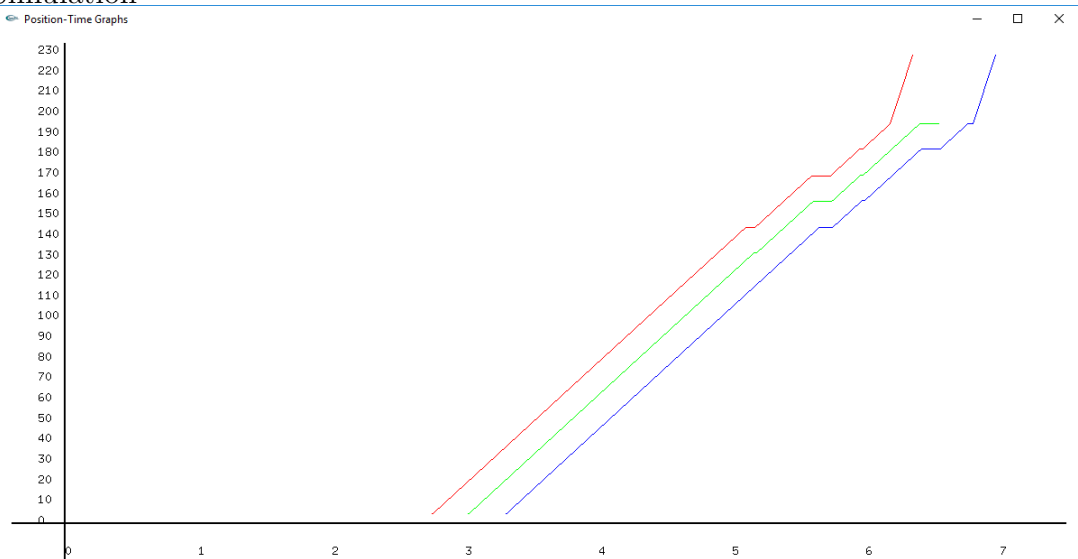

Figure 4.4: Position Time Graph for the 11th, 12th and 13th Vehicles in the Simulation

In this case, the behaviour of 3 consecutive vehicles is shown. The front vehicle (red car) is a consequence of the first vehicle having followed another vehicle (vehicle 10). We see that, after the vehicle drove for 2 seconds, it seized moving and then drove for some 0.5 seconds stopped again, then moved again, stopped for a while and then proceeded to the junction after which it went straight almost immediately after it reached the junction.

The second vehicle replicates the behaviour until it turns at the junction. Contrary, the 13th vehicle, does not immediately replicate the behaviour of the preceding vehicle (12th) although it accelerates with respect to the behaviour of the preceding vehicle. This shows that the vehicle doesn't blindly follow the preceding vehicle i.e. it first checks if the gap between itself and the preceding vehicle is less than the safe distance before it stops. When the 12th vehicle stops for the 2nd time, the 13th vehicle stops too and proceeds to according the behaviour of the preceding vehicle but as when it reaches the junction, it also stops momentarily even though the preceding vehicle had already turned. This implies that the stopping was as result of presence of another turning vehicle hence it had to give way.

4.3 Scenario 3: Position-Time Snapshot and Graph for 10 vehicles in the Simulation

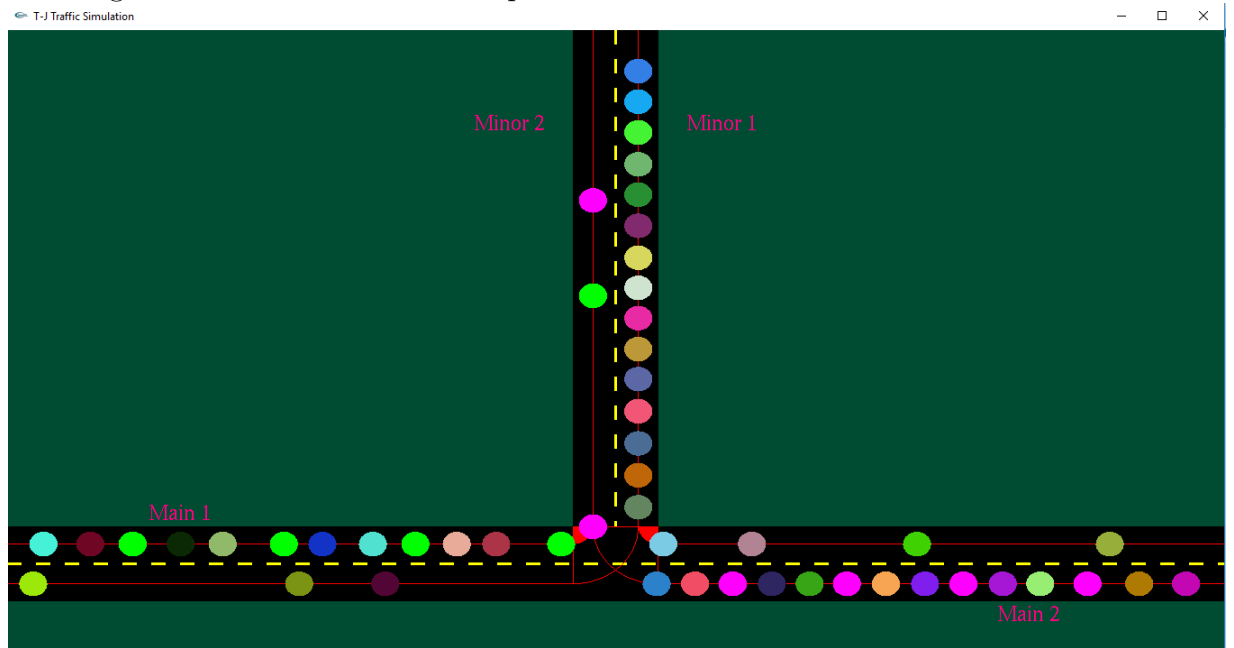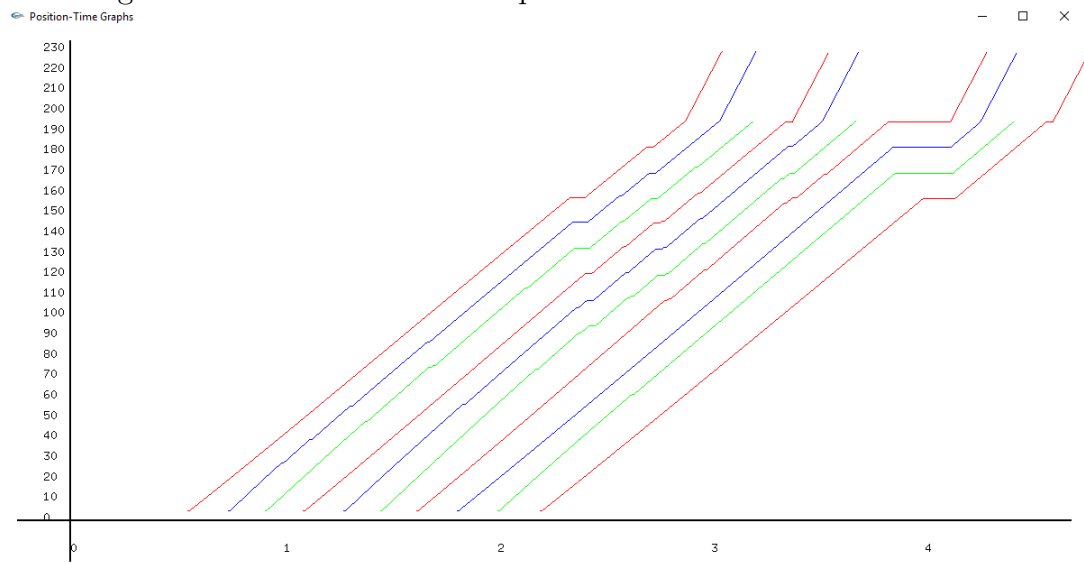Figure 4.5: Position-Time Snapshot for 10 vehicles in the Simulation

Figure 4.6: Position-Time Graph for 10 vehicles in the Simulation

This graph illustrates the exact sequence of propagation of the behaviour of the preceding vehicle especially in the case where jam has occurred. Since there is a

high density of vehicles, when the preceding vehicle stops for some time, it causes five more vehicles to stop. The 7th vehicle is affected by the jam propagation when the condition gap is greater than safe distance becomes false. This is the same case for the 8th, 9th and 10th vehicle. It is therefore concluded that in so long as the gap between the preceding and the following vehicles is greater than the safe distance, the vehicle will always accelerate.

4.4 Scenario 4: Traffic Density-Time Snapshot and Graph for the Traffic Simulation

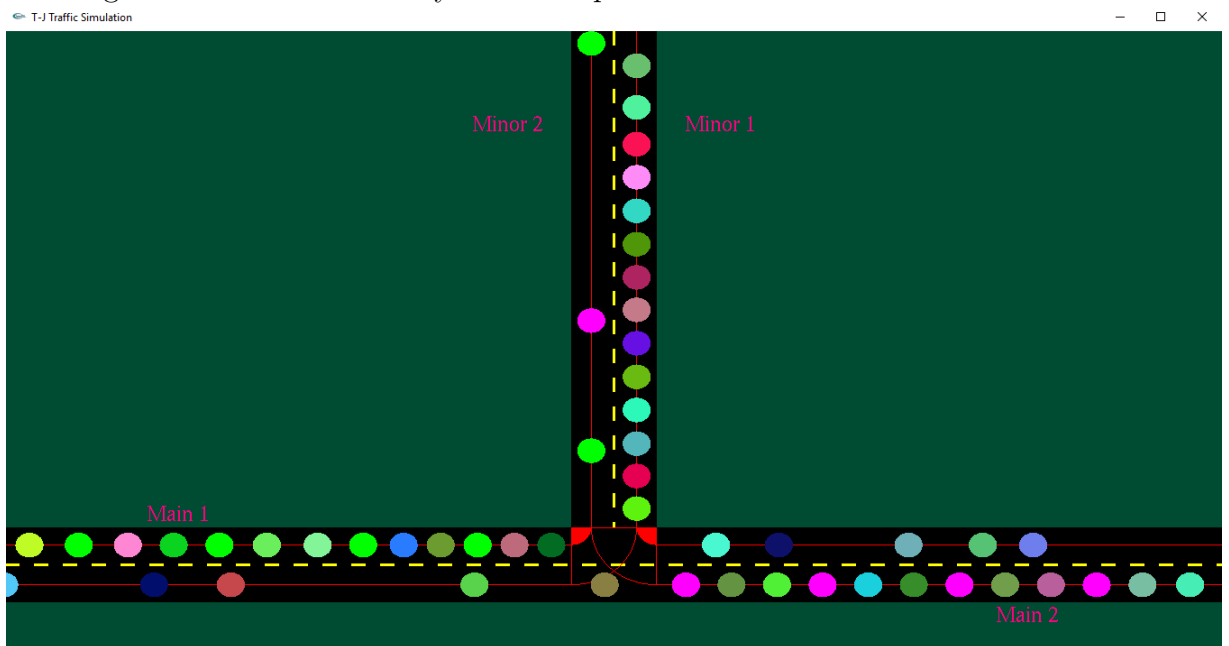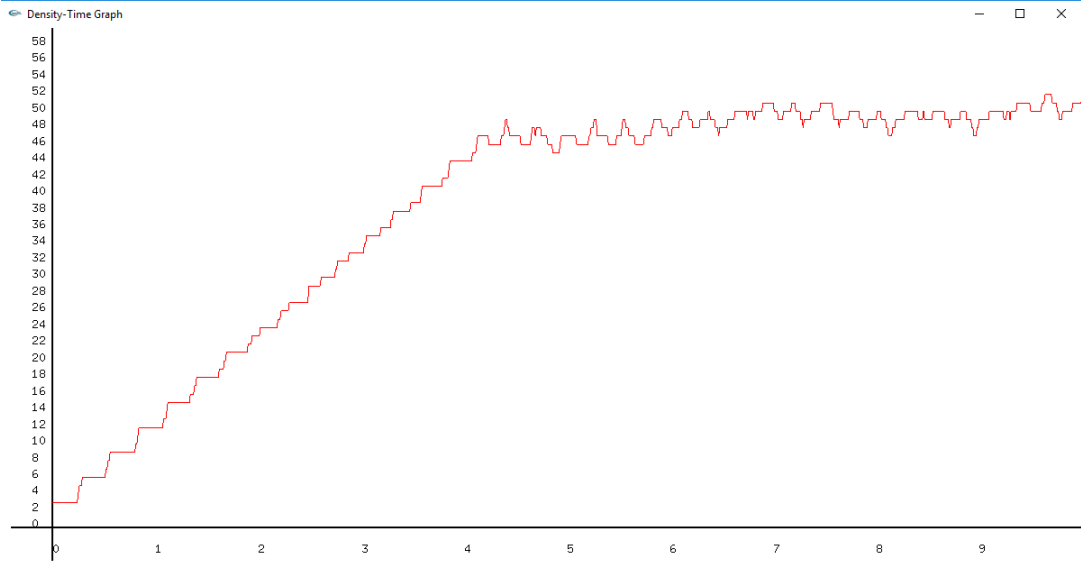Figure 4.7:   Traffic Density-Time Snapshot for the Traffic Simulation

Figure 4.8:   Traffic Density-Time Graph for the Traffic Simulation

This graph illustrates how a jam draws around a T-Junction. In the graph, it is seen that before the simulation starts there are two vehicles on the road. As more vehicles are generated the density of the traffic on the road grows gradually over time. In some instances, the density remains constant when there's a balance between the incoming and outgoing vehicles. As more and more vehicles are generated, the density increases until all the lanes are jammed. By the use of the Stop and Go Condition that is based on prioritization, the density fluctuates as vehicles leave and enter the junction respectively but the density is maintained at interval $45 - 55$. This explicitly means that the jam will never end.

The Theoretical implications of this model are its ability for use by traffic designers in studying how to design and prioritize different aspects in the road. Practically, it can be used when trying to automize systems in the road. Its biggest advantage is its ability to be used in all scenarios both automized and non-automized. The model has been tested through observation and the results agree with the works by Mathew (2014); Gunay (2007); Helbing et al. (2002) which makes it valid.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

In this chapter the results of the simulation presented in Chapter 4 are discussed. This is done by using the data produced during the simulation process.

5.1 Conclusion

A Microscopic Traffic Flow model that incorporates lane changing was analyzed. A qualitatively comprehensive study done reveals that intersections are a huge contributor to traffic jams if not well managed. The Methodology used gives directions and practical applications in no-automated traffic ways. Using the Cauchy Lipschitz Theorem, existence and uniqueness of results was also determined.

The result analysis reveals that as long as all conditions are adhered to to the latter, traffic flow will be stable. However, during the peak times, there the tendency of an influx in traffic with more vehicles coming on the road. This, in our simulation is demonstrated by reducing the car generation time so that many cars are generated and released to the traffic consideration realm after a very short time. The, results obtained are in total agreement with what happens in real world.

Based on the simulation output, it is seen that vehicles were able to change lanes by the use of the Turning and Prioritization Logic. This implies an extension of the previously existent GM model. Also, the simulation clearly depicted the formation jams at unsignalised intersections since the vehicles depended entirely on the priority to turn. Therefore, if a vehicle had a lower priority to turn, it was forced to wait until no higher ranked conflicting turns. Using the GM Model, the stopping is always propagated to the following vehicles which leads to the gradual

growth of traffic jam over time.

Additionally, the simulation shows that the higher the supply of vehicles into the lanes the faster the growth of jams. Further, if the vehicles move at low speeds, jam development rate is higher and the contrary is always true. It was also concluded that a jam may result due to a timid driver who, instead of driving off the junction, intentionally or unintentionally stops even after entering the intersection when all the conditions are favorable (when he or she should actually exit the intersection).

Finally, it is established that for safe merging, several strategies must be employed; prioritization where vehicles moving straight and or turning right from the main road/lane into a minor road or lane are given the highest priority. Second in that order are those vehicle turning left from the main or turning right into the main road. The lowest priority is given to those turning left into the main road. Maintaining a Safe Distance is also another between the preceding and the following vehicle is another very impressive strategy for safe merging. A vehicle should also leave the junction at a higher velocity if it enters so that it gives way to the others.

5.2   Recommendations

(i) Based on the findings, when designing roads, engineers should adopt prioritization strategies or construct roads such that there'll not be collisions at the junctions

(ii) Based on the findings by this model simulation, the minimum acceptable speed should be established based on an informed decision that highly depends on various other factors such as use of the road by other people, the level of effects of jams occurring at that particular junction etc. This decision should not only check at the speed while inside the junction but also as drivers approach the junction.

(iii) Further, based on the findings, timing of the different lanes to allow vehicles in different traffic lanes and the appropriate safe gap determination should be done decisively such that no lane is starved waiting for other lanes to give turning priority. This is achieved by ensuring that the Main/Major roads are allocated more time than the Minor ones at all times unless need dictates otherwise.

5.2.1   Future Work

(i) Improve the numerical part by introducing Runge-Kutta Method and compare to see if the results will be refined.

(ii) Do the same study with several junctions and combine main road and round about(s) by putting in some points/area traffic signals to do the control

(iii) Do the study by introducing more control strategies.

REFERENCES

Ahmed, K. I. (1999). *Modeling drivers' acceleration and lane changing behavior.* PhD thesis, Massachusetts Institute of Technology.

Borsche, R., Kimathi, M., and Klar, A. (2012). A class of multi-phase traffic theories for microscopic, kinetic and continuum traffic models. *Computers and Mathematics with Applications*, 64(9):2939–2953.

Chandler, R. E., Herman, R., and Montroll, E. W. (1958). Traffic dynamics: studies in car following. *Operations research*, 6(2):165–184.

Chen, C., Wang, Y., Li, L., Hu, J., and Zhang, Z. (2012). The retrieval of intra-day trend and its influence on traffic prediction. *Transportation research part C: emerging technologies*, 22:103–118.

Clement, S., Vogiatzis, N., Daniel, M., Wilson, S., and Clegg, R. (2014). Road networks of the future: can data networks help? In *28th Australasian Transport Research Forum*.

Delis, A., Nikolos, I., and Papageorgiou, M. (2014). High-resolution numerical relaxation approximations to second-order macroscopic traffic flow models. *Transportation Research Part C: Emerging Technologies*, 44:318–349.

Edie, L. C. (1961). Car-following and steady-state theory for noncongested traffic. *Operations Research*, 9(1):66–76.

Gazis, D. C., Herman, R., and Potts, R. B. (1959). Car-following theory of steady-state traffic flow. *Operations research*, 7(4):499–505.

Gazis, D. C., Herman, R., and Rothery, R. W. (1961). Nonlinear follow-the-leader models of traffic flow. *Operations research*, 9(4):545–567.

Gipps, P. G. (1986). A model for the structure of lane-changing decisions. *Transportation Research Part B: Methodological*, 20(5):403–414.

Goerigk, M., Schachtebeck, M., and Schobel, A. (2013). Evaluating line concepts using travel times and robustness. *Public Transport*, 5(3):267–284.

Gunay, B. (2007). Car following theory with lateral discomfort. *Transportation Research Part B: Methodological*, 41(7):722–735.

Helbing, D., Hennecke, A., Shvetsov, V., and Treiber, M. (2002). Micro-and macro-simulation of freeway traffic. *Mathematical and computer modelling*, 35(5):517–547.

Hidas, P. (2002). Modelling lane changing and merging in microscopic traffic simulation. *Transportation Research Part C: Emerging Technologies*, 10(5):351–371.

Hidas, P. (2005). Modelling vehicle interactions in microscopic simulation of merging and weaving. *Transportation Research Part C: Emerging Technologies*, 13(1):37–62.

Hidas, P. and Behbahanizadeh, K. (1999). Modelling traffic incidents with sitras. In *International Symposium on Automotive Technology 'I&' Automation (32nd: 1999: Vienna, Austria). Surface transportation advances and intelligent transportation systems*.

Huang, S. and Sadek, A. W. (2009). A novel forecasting approach inspired by human memory: the example of short-term traffic volume forecasting. *Transportation Research Part C: Emerging Technologies*, 17(5):510–525.

Hunt, J. and Yousif, S. (1990). Merging behaviour at roadworks. In *PTRC Summer Annual Meeting, 18th, 1990, University of Sussex, United Kingdom*.

Kerner, B. S. (2009). Traffic congestion, modeling approaches to. In *Encyclopedia of Complexity and Systems Science*, pages 9302–9355. Springer.

Kolen, H. (2013). *Modelling merging behaviour on freeway on-ramps*. PhD thesis, TU Delft, Delft University of Technology.

Kou, C. and Machemehl, R. B. (1997). Modeling vehicle acceleration deceleration behavior during merge maneuvers. *Canadian journal of civil engineering*, 24(3):350–358.

Lo, H. K., Lam, W. H., Wong, S. C., and Leung, J. M. (2011). Advanced systems for public transport scheduling and network design. *Public Transport*, 3(1):1–2.

Loot, M. (2009). *Empirical analysis of merging behaviour: A microscopic approach to establish a new theory*. PhD thesis, TU Delft, Delft University of Technology.

Lu, X.-Y., Tan, H.-S., Shladover, S. E., and Hedrick, J. K. (2000). Implementation of longitudinal control algorithm for vehicle merging. *Ann Arbor*.

Luspay, T., Kulcsar, B., Varga, I., Zegeye, S., De Schutter, B., and Verhaegen, M. (2010). On acceleration of traffic flow. In *Intelligent Transportation Systems (ITSC), 2010 13th International IEEE Conference on*, pages 741–746. IEEE.

Mathew, T. V. (2014). Lane changing models. *Transportation systems engineering anonymous*, pages 15–1.

McDonald, M., Wu, J., and Brackstone, M. (1997). Development of a fuzzy logic based microscopic motorway simulation model. In *IEEE Conference on Intelligent Transportation System*, pages 82–87.

Ntousakis, I. A., Porfyri, K., Nikolos, I. K., and Papageorgiou, M. (2014). Assessing the impact of a cooperative merging system on highway traffic using a

microscopic flow simulator. In *ASME 2014 International Mechanical Engineering Congress and Exposition*, pages V012T15A024–V012T15A024. American Society of Mechanical Engineers.

Prasetijo, J. (2007). *Capacity and traffic performance of unsignalized intersection under mixed traffic condition*. PhD thesis, Citeseer.

Raslaviius, L., Kerys, A., Pukalskas, S., Bazaras, J., Jablonskyt, J., Ilgakojyt-Bazarien, J., and Makaras, R. (2015). City transport analysis using the general motors (gm) microscopic model. *Public Transport*, 7(2):159–183.

Robert, H. and B, P. R. (1961). Single lane traffic theory and experiment.

Ruskin, H. J. and Wang, R. (2002). Modeling traffic flow at an urban unsignalized intersection. In *International Conference on Computational Science*, pages 381–390. Springer.

Systematics, C. (2005). Traffic congestion and reliability: Trends and advanced strategies for congestion mitigation. *Final Report, Texas Transportation Institute.*

Wang, J. (2005). A simulation model for motorway merging behaviour. *Transportation and traffic theory*, 16:281–301.

Yang, Q. and Koutsopoulos, H. N. (1996). A microscopic traffic simulator for evaluation of dynamic traffic management systems. *Transportation Research Part C: Emerging Technologies*, 4(3):113–129.

Zhang, H. M. and Kim, T. (2005). A car-following theory for multiphase vehicular traffic flow. *Transportation Research Part B: Methodological*, 39(5):385–399.

Zhang, Y. (2004). *Scalability of car-following and lane-changing models in microscopic traffic simulation systems*. PhD thesis, Faculty of the Louisiana State

University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering in The Department of Civil and Environmental Engineering by Yan Zhang BS, Southeast University.