MODELING FLUID FLOW IN AN OPEN RECTANGULAR CHANNEL WITH LATERAL INFLOW CHANNEL

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DECLARATION

This thesis is my original work and has not been submitted to any other university for examination.

Signature:.................................. Date:.........................

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This thesis report has been submitted for examination with our approval as University supervisors.

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Prof. Mathew Kinyanjui (JKUAT, KENYA)

DEDICATION

This thesis is dedicated to my late brother Boniface Ndung'u Karimi, his wife Katrina Ndung'u and his lovely daughter Ashley Njeri. Rest in peace my brother and friend.

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ABSTRACT

In this study, we investigate the flow in an open rectangular channel with a lateral inflow channel. The flow of an incompressible Newtonian fluid through a man-made open rectangular channel with a lateral inflow channel is investigated. We have considered the effects of angle as it varies from zero to ninety degrees, the cross-sectional area, velocity and length of the lateral inflow channel on how they affect the flow velocity in the main open rectangular channel. Since the discharge is directly proportional to the flow velocity, the increase in the flow velocity means an increase in the discharge and vice versa. The equations governing the flow are the continuity and momentum equation of motions, which are highly nonlinear and cannot be solved by an exact method. Therefore, an approximate solution of these partial differential equations is determined numerically using the finite difference method. The finite difference method is used to solve these equations because of its accuracy, consistency, stability and convergence.

Matlab software is used to generate the results which are then analyzed using graphs. The findings are that, at zero degrees of the lateral rectangular channel, the results compare to earlier research done. It is also found out that an increase in the area and the length of the lateral inflow channel leads to a reduction in the velocity while an increase in the velocity of this channel leads to an increase in the velocity of the main channel. Finally, an increase in the angle of the lateral inflow channel does not necessarily lead to an increase in the velocity in the main channel. That, angles of between 30^0 and 50^0 exhibits higher values of velocities in the main open channel compared to other angles of the lateral inflow channel.

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NOMENCLATURE

Symbol	**Meaning**
A	Cross-sectional area of flow of the main channel (m^2)
a	Cross-sectional area of flow of the lateral inflow channel (m^2)
b	Top width of the main channel (m)
$b1$	Top width of the lateral inflow channel (m)
C	Resistance coefficient of flow (Chezy coefficient)
y	Depth of flow in the main channel (m)
$y1$	Depth of flow in the lateral inflow channel (m)
K	constant of proportionality in Chezy formula
L	Length of the lateral inflow channel (m)
P	Wetted perimeter of the main channel (m)
Q	Discharge in the main channel ($m^3\ s^{-1}$)
R	Hydraulic radius (m)
Re	The Reynolds number
F_r	The Froude number
F_R	Resisting force (N)
S_o	Slope of the channel bottom (m)
S_f	Friction slope (m)
T	Top width of the free surface (m)
$\vec{T}$	Traction force (N)
V	Mean velocity of flow in the main channel (ms^{-1})
u	Mean velocity of flow in the lateral inflow channel (ms^{-1})
g	Acceleration due to gravity (ms^{-2})
n/ni	The manning coefficient of roughness ($s\ m^{-1/3}$)

q	Lateral discharge or uniform inflow (m^3s^{-1})
t	Time (s)
x	Distance along the main flow direction (m)
$\vec{F}$	Body force (N)
W	Weight of the fluid (N)
$\vec{q}$	Velocity vector ($ui + vj + wk$)
Δ	Forward difference
∇	Divergence vector
α	Energy coefficient
ν	Kinematic viscosity (m^2s)
γ	Fluid specific weight (N)
τ	Shear stresses (Nm^{-2})
ρ	Density($kg\ m^{-3}$)
ϑ	Coefficient of viscosity (Ns^{-1})
θ	Angle of lateral discharge channel (degrees)

CHAPTER ONE

INTRODUCTION

1.1 Background information

In the year 2002, Kenya experienced heavy rainfall, which resulted in bridges being swept away as rivers flooded. During this heavy rainfall, water flowed from highlands to lowlands and led to saturation of the soil, which resulted in excess water remaining stagnant. Moreover, some areas still suffer from floods even when normal rain falls. Thus, designing channels that would control such an environmental disaster and more so divert the same water to agricultural land is very important. The fact that the flood problem still persists and the need to convey water for irrigation is still in demand, there is need to come up with an efficient model of a channel with lateral inflow channel to convey the maximum discharge.

A channel may be closed or open at the top. The channels that have an open top are referred to as open channels while those with a closed top are referred to as closed conduits. Good examples of open channels are rivers and streams while examples of closed conduits are pipes and tunnels. . In the past, open channels made of earth and concrete have been designed to meet these needs. They have been of different cross-sections such as trapezoidal, rectangular and circular.

This chapter begins with some definitions of open channel terminologies followed by literature review related to the study of open channels with lateral inflow channel. Finally, the model of the problem, objectives, research questions, null hypothesis and justification of the research are presented towards the end of the chapter.

1.2 Definitions

In this study several terms will be used extensively and in this section such terms are defined. These terms are mostly used in channel flow.

1.2.1 Fluid

Matter is said to be a fluid if it undergoes continuous deformation when some external force is applied. It is said to undergo deformation if the distance between any two neighboring molecules change. A fluid has no definite shape but assumes the shape of the container. Fluids are conventionally classified as liquids or gases. Liquids do not change significantly in volume when subjected to change in pressure and temperature. For this reason they are treated as incompressible fluids. Gases show notable volume changes when subjected to change in pressure and temperature. This implies that they are compressible.

1.2.2 Newtonian fluid

Gutfinger and Pnueli (1992), defined that a fluid as Newtonian if it obeys the Newton’s law of viscosity, which states that the shear stress is proportional to the velocity gradient, and the coefficient of viscosity is taken as a constant. Otherwise, if the coefficient of viscosity varies from one point to another in the channel, the fluid is known as non-Newtonian. Since the fluid is considered to be Newtonian, the traction forces in the momentum equation are a sum of the pressure gradient and viscous forces (which are assumed to be uniform throughout the flow). However, we know that no natural free flowing fluid is Newtonian. This is because as the fluid flows in layers, the distance between the layers at different points gradually changes, which makes the coefficient of viscosity to change. In addition, as the fluid moves from one section of the channel due to either variation of the slope or width of the channel due to erosion, the velocity of the fluid is bound to change which will affect the viscosity of the fluid. So for the fluid to be considered as Newtonian is an assumption. The coefficient of viscosity is defined as the shear stress multiplied by the distance between the two adjacent layers of the fluid, and then divided by the relative change in velocity between the two layers.

1.2.3 Open channel flow

The flow of a liquid, for example, water in a conduit may either be in an open channel or pipe flow. The two kinds of flow are similar in many ways but differ in one important aspect. Open channel flow is characterized by a free surface, whereas pipe flow has none. A free surface is defined as the surface of contact between the liquid and the overlying gaseous fluid. According to Chow (1973), in an open channel a fluid does not fill the conduit completely. Flow in open channels is due to the difference in the potential energy. A lateral inflow channel has discharge resulting from the addition of fluid or water along the direction of flow. The channel that adds water to a stream, river, or lake is referred to as a lateral inflow channel while the one that draws water from the latter is referred to as a lateral outflow channel. Lateral inflow may also include groundwater flow and overland flow.

1.3 Classification of flow

There are several types of flows classified according to changes in flow depth with respect to time and space. The flow is said to be steady if the depth of flow at a particular point does not change with time interval under consideration. A flow in which depth changes with time and space is said to be unsteady. This is the most common type of flow and requires the solution of the energy, momentum and friction equations with time. Open channel flow is said to be uniform if the depth and velocity of flow are the same at every section of the channel. Hence it follows that uniform flow can only occur in prismatic channels. For steady, uniform flow, depth and velocity is constant as you move along the channel.

This constitutes the fundamental type of flow in an open channel. It occurs when gravitational forces are in equilibrium with resistance forces. A flow in which depth varies with distance, but not with time is called steady non-uniform flow. The type of flow may either be gradually varied or rapidly varied. The former requires application of energy and frictional resistance equations.

1.4 Types of channels

There are two types of open channels, namely natural and artificial channels. Artificial channels are channels made by man. They include irrigation canals, navigation canals, spillways, sewers, culverts and drainage ditches. They are usually constructed in a regular cross-section shape throughout and are thus prismatic channels. In the field, they are commonly constructed of concrete and have the surface roughness reasonably well defined although this may change with age. Analysis of flow in such channels will give reasonably accurate results.

Natural channels are not regular or prismatic and their materials of construction can vary widely. The surface roughness will often change with time, distance and elevation. Consequently, it becomes more difficult to accurately analyze and obtain satisfactory results for natural channels than it is with man-made ones. This situation may be further complicated if the boundary is not fixed due to erosion and deposition of sediments occur. For analysis, various geometric properties of the channel cross-sections are required. For artificial channels, these can usually be defined using simple geometric equations given the depth of flow.

1.5 State of flow

The effects of viscosity in relation to the inertia forces of the flow govern the state or behavior of open channel flow. The flow may be laminar, transitional or turbulent depending on the effect of viscosity relative to inertia forces. In laminar flow the fluid particles appear to move in thin layers of fluid which seem to slide over adjacent layers with no disruptions between the layers. The flow is turbulent if the inertial forces are strong relative to the viscous forces. In turbulent flow, the fluid particles move in irregular paths. A flow is termed transitional if it is neither laminar nor turbulent.

Reynolds's number is a non-dimensional parameter which represents the effect of viscosity relative to inertia. It is defined as $Re = VL / \nu$, where ν is the kinematic viscosity, V is the mean velocity of flow and L is the characteristic length. The flow in the channel changes from laminar to turbulent if depending on the Reynolds number. If Re is less than about 2000 the flow is laminar and if Re is greater than 4000 the flow is turbulent. The flow is in transition if it is between these values. Laminar flow is known to exist where thin sheets of water flow or where the conditions are altered like in model testing.

The dimensionless Froude number is important in analyzing the effect of gravity in fluid flow. It is defined as the ratio of inertial forces and gravity forces. It is defined as $Fr = V /D$ where D is the hydraulic depth; V is the mean velocity and g is the acceleration due to gravity. If Fr is number one, it means that the inertial forces and gravity are equivalent and critical flow exists. Flow around or at critical is characterized by instability since small changes in the hydraulic condition results to abnormal changes in velocity and depth. If Fr has a value less than one it means that gravity forces dominate and the open channel is classified as sub-critical or tranquil range of flow. If Fr has a value greater than one, it means that the inertial forces dominate and the flow is classified as super-critical flow.

1.6 Statement of the problem

Several studies of fluid flow through open channels have been carried out in the laboratory. However, mathematical modelling of open channels with lateral inflow channel has received little attention. This research seeks to determine the effect of angle, velocity, cross-sectional area and length of the lateral inflow channel on the flow velocity in the open rectangular channel. The fluid to be considered is Newtonian and the flow is uniform. This study, therefore, aims at coming up with a hydraulically efficient model of flow through an open rectangular channel with a lateral inflow channel.

1.7 Geometry of the problem

In the present study, the lateral inflow channel is introduced into the open rectangular channel as illustrated in Fig. 1. The discharge in the open rectangular channel and the lateral inflow channel are denoted by Q and q respectively. L and θ represent the length and the varying angle respectively, of the lateral inflow channel. The net volume of fluid that enters through the cell dx is considered at a time interval dt.

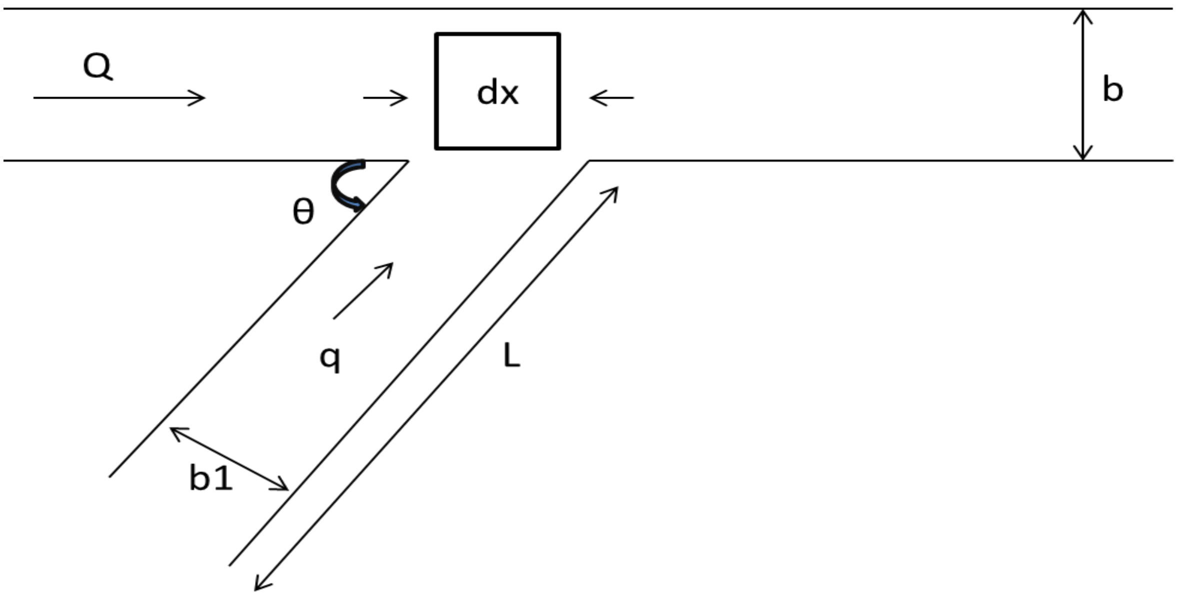

Figure 1: Model of the open rectangular channel with lateral inflow channel at an angle

1.8 Objectives of study

1.8.1 General objective

To analyze fluid flow in an open rectangular channel with a lateral inflow channel

1.8.2 Specific objectives

1. To investigate how the variation of the angle of the lateral inflow channel affects velocity in the main open channel.

2. To investigate how the cross-sectional area of lateral inflow channel affects velocity in the main open channel.
3. To investigate how the length of the lateral inflow channel affects velocity in the main channel.
4. To investigate how the velocity of the lateral inflow channel affects velocity in the main channel.

1.9 Research questions

1. How does the variation of the angle of the lateral inflow channel affect the velocity in the main open channel?
2. How does the cross-sectional area of lateral inflow channel affect velocity in the main open channel?
3. How does the length of the lateral inflow channel affect velocity in the main channel?
4. How does the velocity of the lateral inflow channel affect velocity in the main channel?

1.10 Null hypothesis

The variation of the cross-section area, length, velocity and angle of the lateral inflow channel do not affect the velocity in the main open channel.

1.11 Justification

For any civilization to exist and thrive, it needs water. Water is life, yet too much water can lead to death due to floods. To direct water to lakes and rivers, man has constructed channels and canals. However, the problem of flood still persists, especially when there is heavy rain. Up to date, there is still a challenge to construct a channel that has a lateral inflow channel that will convey the maximum amount of water in an efficient way. Therefore, an efficient model of open channels with lateral intake channels has to be designed to meet these needs.

The mathematical model in this study can be employed in the construction of lateral inflow channels that will increase the discharge while conveying water to farms for irrigation and in draining water from flood stricken areas. Moreover, the findings are applicable in flour or textile production and in the design of water mills where large volumes of high velocity water are required to turn large turbines and also drive mechanical processes.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

The literature review in this chapter deals with three aspects of the current study. In the first section of this review, the equations that govern open channel flow are discussed, followed by review of lateral flow studies and equations. Then the literature on one of the numerical methods that is used to solve open channel flow is discussed and finally a summary why there is a gap in this literature to conduct the present study.

An open channel, whether in the form of an artificial or man-made channel constructed with a view to convey water to the required destination is common in many places. The forces at work in open channels are the inertia, gravity and viscosity forces. By neglecting forces that are generated in the junction area, the momentum equation is used to analyze lateral inflow in an open channel.

2.2 Review of Open channel flow equations

Chezy equation was one of the earliest equations developed for average computations of the velocity of a uniform flow, Henderson (1966). However, the formula that is mostly used in open channel problems is the Manning formula, Bilgil (1998). The Manning formula is highly very useful compared to the Chezy equation because it takes into account the degree of the channel irregularity, the bed materials, the relative effect of obstruction of the channel, vegetation growing in the channel, variation in shape and size of the channel and meandering in the channel, Chadwick and Morfet (1993).

Many relationships, such as the velocity formula for open channel flow were studied by Chow (1959), which helped in analyzing open channels. The principles of conservation of momentum and mass are preserved by the Saint Venant equations and that is why they are used to govern unsteady open channel flow. The mathematical models of these unsteady open channel flows are applicable in flood defense and irrigation control design, Chaudhry (1993). Tuiteek and Hicks (2001), with the aim of controlling floods were able to model unsteady flow in compound channels. They incorporated some terms to account for the momentum transfer phenomenon to incorporate unsteady flow in compound channels by developing a model based on the Saint Venant equations of flow.

Kwanza, Kinyanjui and Nkoroi (2007) studied and analyzed the effects of slope of the channel, the width of the channel, velocity and depth as they vary from one point to another in the channel and the lateral discharge on the fluid velocity and channel discharge for both rectangular and trapezoidal channels. They noted that to increase the discharge in the channels, the slope of the channel, width of the channel and the lateral discharge need to be increased. In addition, by minimizing the wetted perimeter, the velocity of the fluid flow increased. Thiong'o (2011) investigated fluid flow in open rectangular and triangular channels. Her findings on the study of rectangular channels compared with Kwanza *et al* (2007). That, the velocity in the open rectangular channel increased when the slope, discharge and width increased. However, increasing the wetted perimeter of the channel resulted in a decrease in the flow velocity. To solve the continuity and momentum equations, they both used the finite difference method as a numerical tool.

2.3 Review of flow with Lateral inflow flow

Taylor (1944) presented the earliest literature on open-channel junction flows showing that the number of equations provided by a one dimensional analysis based on the momentum conservation is incomplete to solve analytically the junction problem, Kesserwani (2008a, 2008b). Later, the branch channel problem was regarded as a lateral overflow through a side-weir of zero crest height, Rajaratnam and Pattabhiramiah (1960).

The outflow and inflow discharge, the downstream and upstream and depth of water and the recirculation flow in the branch channel characterizes open channel dividing flow. Ramamurthy and Satish (1988), and Ingle and Mahankal (1990) were all able to establish that the downstream to the upstream discharge ratio of the main channel was the most relevant parameter that analyzed open flow with a lateral channel at 90^0. Comparing these results with some experimental observations, it was observed that the results of the above analysis were satisfactory. Neary and Odgaard (1993) also concluded that the roughness of the bed as well as the branch-channel to main-channel velocity-ratio would affect the structure of the flow. Barkdoll (1999) was able to demonstrate in his research that the diversion flow ratio has the greatest effect on the sediment delivery ratio of the lateral intake, which is carried out in a straight path with 90^0 intake angle. Ramamurthy (2007) presented experimental data for open channels with dividing flows that were related to three dimension main velocity components and water surface profiles. Yang (2009) was able to study flow structures with diversion angles of 90°, 45° and 30°. A diversion angle of between 30° and 45° was recommended to get a better flow pattern of the fluid. Fan and Li (2005) were able to formulate the diffusive wave equations for open channel flows with uniform and concentrated lateral inflow. In their formulation, they were able to present the continuity and momentum equations of an open channel with a lateral inflow channel that joins the main open channel at a varying angle.

Ramamurthy and Satish (1988) theoretically and experimentally investigated dividing flows with a submerged lateral branch when focusing on the sub-critical flow regime. The investigators developed a model theoretically by relating the discharge ratios and the downstream-to-upstream depth with the upstream Froude number. Ramamurthy (1990) formulated a more general expression with no restriction on the flow nature of the lateral branch. The best that can be expected is an approximation of the theoretical relationship linking flow rate ratio and depth ratios of upstream and downstream of the junction for sub-critical flow divisions. The discharge in the branch of the lateral channel can be computed by the formula formulated by Mizumura (2003) for super-critical overflowing rivers, which, when compared with experiments by Mizumura (2003 and 2005) compares well.

Mohammed (2013) investigated how the discharge coefficient is affected by varying four different angles using an oblique weir with respect to the side of the channel wall in the flow direction. The four angles were 30^o, 60^o, 75^o and 90^o all of which were varied along the flow direction. The findings were that maximum discharge was achieved at angle 30^0 of the side weir. Moreover, Masjedi and Taeedi (2011) studied in the laboratory the effect of intake angle on discharge ratio in lateral intakes in 180^0 bend. The investigations were carried out in a laboratory flume under clear water. The experiments were conducted with varying Froude number of various intake angles. The investigations showed that the discharge ratio increased at a lateral intake angle of 45^0 in all locations of the 180^0 flume bend.

2.4 Review of the numerical method

Shamaa (2002) used the finite difference Preissmann implicit model to solve open channel operation-type problems which were based on the Saint Venant equations. Comparing with an explicit model, the implicit finite difference method model showed less oscillation and more accuracy.

Akbari and Firoozi (2010) investigated the Preissmann and Lax diffusive schemes which are two different numerical methods for the numerical solution of the Saint Venant equations that govern the propagation of flood wave in natural rivers with the objective of gaining better understanding of the propagation process. These findings in the flood wave propagations showed that the hydraulic parameters play an important role in these waves. Chagas and Souza (2005) provided the solution of Saint Venant equation through the study of flood in rivers by discretization for better understanding of this propagation process. Their findings in the propagation of the flood wave showed that the hydraulic parameters play an important role in these waves.

2.5 Summary

The Saint Venant equations are used to analyze fluid flow in both open channels and lateral inflow channel. The finite difference method is then used as a numerical tool to solve the equations. The literature above demonstrates that much research seems to have been done in open channels with no lateral inflow channel. However, the research that has been done on the lateral inflow channel is limited only in the laboratory. Therefore, little research has been done in open channels with lateral inflow channel by modeling the problem mathematically. That is why in our research, we shall model our problem mathematically using the Saint Venant equations and use the finite difference method to solve the equations.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

In this chapter, we review some uniform flow formulas like the Chezy and Manning formulae that have been developed over the years. Next the saint Venant equations which govern open channel flow are outlined and modified to incorporate an open channel with the lateral discharge at an angle. Then, the method of solution is also discussed. Moreover, the governing equations in their finite difference forms with the initial and boundary conditions are presented. Finally, investigations are done on how the variation of the angle, the cross-sectional area, velocity and length of the lateral inflow channel affect the velocity in the main rectangular channel.

3.2 Assumptions

1. The flow is one-dimensional such that the main component of velocity is along the x-axis and is a function of x alone
2. The forces causing the flow are due to gravity alone
3. The fluid is considered incompressible
4. The fluid is Newtonian
5. The flow is unsteady
6. Sediment formation between the lateral inflow channel and the main open channel is negligible
7. Turbulent formation between the lateral inflow channel and the main open channel is negligible.

3.3 The Chezy and Manning formula

3.3.1 The Chezy formula

This formula is based on two assumptions, Cecen (1982). First, the force resisting the flow per unit of wetted area is proportional to the square of the velocity, and second, the force causing the motion equals to resistance force.

From the first assumption, resisting force per unit of wetted area is proportional to velocity squared, which implies it is equal to KV^2 where K is a constant. Hence

$$F_R = KV^2 \tag{3.1}$$

Wetted area = wetted perimeter (P) multiplied by Length (L) = PL, where

$P = b + 2y$ and total resisting force

$$F_R = KLPV^2 \tag{3.2}$$

Force that is causing the flow is equal to the component of the weight of water in the direction of flow or

$$= VALsin\theta \tag{3.3}$$

where θ is the angle of inclination of the channel bottom with the horizontal force causing flow.

By the second assumption

$$KLPV^2 = \gamma ALS, \text{ which implies} \tag{3.4}$$

$$V = \sqrt{\frac{\gamma}{K}}\sqrt{\frac{A}{P}}\sqrt{S} = C\sqrt{RS}, \text{ where} \tag{3.5}$$

$$R = \frac{A}{P}, \text{ known as the hydraulic radius,} \tag{3.6}$$

$$C = \frac{\gamma}{K}, \text{ is the flow resistance factor known as the Chezy's coefficient} \tag{3.7}$$

This coefficient is believed to be dependent upon the channel slope S, the hydraulic radius R and the coefficient of roughness n.

3.3.2 The Manning formula

The formula was developed by Bilgil (1998) through studies he performed. Then the manning formula is given by,

$$V = \frac{1}{n} R^{\frac{2}{3}} S_f^{\frac{1}{2}} \tag{3.8}$$

According to the above equation, the mean velocity of flow is a function of the hydraulic radius, channel roughness and the energy gradient slope where in uniform flow the energy gradient slope is assumed to be equal to the channel bottom slope.

Given the velocity, the discharge is defined as the product of cross-sectional area and velocity,

$$Q = AV \tag{3.9}$$

Substituting the Manning equation (3.8) into equation (3.9) we get

$$Q = KAR^{\frac{2}{3}}, \text{ where} \tag{3.10}$$

$$K = \frac{1}{n} S_f^{\frac{1}{2}} \tag{3.11}$$

This equation gives the flow rate through a channel of a given slope, radius and roughness coefficient.

From (3.6), R increases as P decreases. Furthermore, from (3.10), an increase in R will lead to increase in the discharge Q.Hence, to get a maximum discharge from our channel, we need to minimize P, which is the wetted perimeter.

We now consider an open rectangular channel with depth y and width b. Then the wetted perimeter is,

$$\mathrm{P} = \mathrm{b} + 2\mathrm{y} \tag{3.12}$$

The cross-section area of the channel is given by

$$A = by \tag{3.13}$$

Now, making b the subject of the formula from the equation (3.13) we have

$b = \frac{A}{y}$ (3.14)

Substituting equation (3.14) into equation (3.12) we get

$P = \frac{A}{y} + 2y$ (3.15)

For a given channel slope S, surface roughness n and area A, the wetted perimeter P, will be maximized when

$\frac{dP}{dy} = 0$ (3.16)

Now, $\frac{dP}{dy} = -\frac{A}{y^2} + 2$ (3.17)

From the condition (3.16) and (3.17), we have

$-\frac{A}{y^2} + 2 = 0$ (3.18)

$\frac{A}{y^2} = 2$ (3.19)

$A = 2\,y^2$ (3.20)

Now to confirm that p is maximized, we check $\frac{d^2P}{d^2y} > 0$

Differentiating equation (3.17) with respect to y we get

$\frac{d^2P}{d^2y} = \frac{2A}{y^3}$ (3.21)

Which means that since A and y are always positive, then from the equation (3.21),

$$\frac{d^2P}{d^2y} > 0$$

Substituting equation (3.13) in equation (3.20), we get

$b = 2y$ (3.22)

Therefore, to achieve maximum discharge for an open rectangular channel, the width b should be twice the depth y.

3.4 Governing equations

The Saint Venant's equations basically describe the propagation of a wave in an open channel, predominantly one dimensional flow where the fluid is incompressible. These equations are the continuity equation and equation of motion derived from Newton's second law of motion. Moreover, according to the number of elements considered in the model, waves can be classified as gravitational waves, diffusive waves, dynamic waves or cinematic waves.

3.4.1 Continuity equation

The principle of continuity is based on the law which states that mass can neither be created nor destroyed. Therefore a continuity equation is a type of differential equation that describes the transport of some kind of conserved quantity for example mass.

For any arbitrary shape, the continuity equation governing unsteady flow in open channels is,

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \qquad (3.23)$$

According to figure 1, the cell with lateral inflow dx, in a time interval dt is considered. The net volume of the fluid in this cell is $\frac{\partial Q}{\partial x}\ dxdt$. But since our lateral discharge channel is inclined at an angle θ, the lateral inflow is $\frac{q}{L}\sin\theta\ dxdt$. The increment of the fluid in this cell is $\frac{\partial A}{\partial t}\ dxdt$. Considering the density of our fluid as constant and in line with the conservation law of the fluid, we have

$$\frac{\partial Q}{\partial x} dx\,dt + \frac{\partial A}{\partial t} dx\,dt = \frac{q}{L}\sin\theta\ dxdt \qquad (3.24)$$

This equation (3.24) reduces to

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = \frac{q}{L}\sin\theta \qquad (3.25)$$

Substituting equation (3.9) above into equation (3.25) and differentiating partially with respect to *x* we get

$$V\frac{\partial A}{\partial x} + A\frac{\partial V}{\partial x} + \frac{\partial A}{\partial t} = \frac{q}{L}\sin\theta \qquad (3.26)$$

The flow area can be assumed to be a known function of the depth and therefore the derivatives of A can be expressed in terms of y.

$$\frac{\partial A}{\partial x} = \frac{dA}{dy}\frac{\partial y}{\partial x} = T\frac{\partial y}{\partial x} \qquad (3.27)$$

$$\frac{\partial A}{\partial t} = \frac{dA}{dy}\frac{\partial y}{\partial t} = T\frac{\partial y}{\partial t} \qquad (3.28)$$

Where T is the channel top width and Franz (1982) assumed that T is determined by

$$T = \frac{dA}{dy} \qquad (3.29)$$

Since the area of the channel is given by $A = Ty$, where T is the top width and y the depth of the channel, we get the equation (3.29).

Substituting equation (3.27) and (3.28) into equation (3.26) we get,

$$VT\frac{\partial y}{\partial x} + A\frac{\partial V}{\partial x} + T\frac{\partial y}{\partial t} = \frac{q}{L}\sin\theta \qquad (3.30)$$

Dividing equation 3.30 throughout by T and rearranging we get,

$$\frac{\partial y}{\partial t}+V\frac{\partial y}{\partial x}+\frac{A}{T}\frac{\partial V}{\partial x}=\frac{q}{T\,L}\sin\theta \qquad (3.31)$$

The term q can be defined by

$$q=ua \qquad (3.32)$$

Furthermore, the area in the lateral intake angle is given by,

$$a=b1.\,y1 \qquad (3.33)$$

Equation (3.31) is the general equation of continuity for open channel flow with lateral inflow channel at an angle.

3.4.2 Momentum equation

Since the energy equation does not account for the dissipation of energy due to the turbulence of the fluid that is generated by the mixing of the open rectangular channel and the lateral inflow channel, the momentum equation becomes an appropriate equation for lateral inflow problems. The momentum equation describes the motion of fluid particles. This equation is derived from the Newton’s second law of motion, together with the assumption that fluid stress is the sum of diffusing viscous term, plus a pressure term. This equation relates the sum of forces acting on an element of fluid to its acceleration or rate of change of momentum.

The law of conservation of momentum requires that the time rate of change of the momentum accumulated within the element is equal to the sum of the net rate of momentum transfer into the element and sum of the external forces in the flow direction.From figure 1, in a time interval of dt, the net momentum for the cell dx is $\frac{\partial(QV)}{\partial x}dxdt$. The lateral inflow component of velocity in the flow direction is $u\cos\theta$. Thus, lateral inflow momentum into cell dx at a time interval dt becomes $\frac{q}{L}\sin\theta\ u\cos\theta\ dxdt$. The fluid pressure and fluid weight in the direction of flow are $g\frac{\partial(yA)}{\partial x}dx\,dt$ and $gA\left(S_f - S_0\right)dxdt$ respectively. The increment in the momentum for the cell dx is $\frac{\partial Q}{\partial t}\,dxdt$ .Therefore, according to the conservation law in the momentum equation we have,

$$\frac{\partial Q}{\partial t}dx\,dt + \frac{\partial(QV)}{\partial x}\,dx\,dt + g\frac{\partial(yA)}{\partial x}dxdt + gA\left(S_f - S_0\right)dxdt$$
$$= \frac{q}{L}\sin\theta\;u\cos\theta\;dx\,dt \qquad (3.34)$$

This equation (3.34) simplifies to

$$\frac{\partial Q}{\partial t} + \frac{\partial(QV)}{\partial x} + g\frac{\partial(yA)}{\partial x} + gA\left(S_f - S_0\right) = \frac{q}{L}\sin\theta\;u\cos\theta \qquad (3.35)$$

Substituting equation (3.9) into equation (3.35) above and differentiating partially with respect to *x* considering the area A is a constant we get,

$$A\frac{\partial V}{\partial t} + V\frac{\partial A}{\partial t} + Q\frac{\partial V}{\partial x} + V\frac{\partial Q}{\partial x} + gA\frac{\partial y}{\partial x} + gA\left(S_f - S_0\right) = \frac{q}{L}\sin\theta\;u\cos\theta \qquad (3.36)$$

Rearranging equation (3.36), we have

$$V\left(\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x}\right) + A\frac{\partial V}{\partial t} + Q\frac{\partial V}{\partial x} + gA\frac{\partial y}{\partial x} + gA\left(S_f - S_0\right) = \frac{q}{L}\sin\theta\;u\cos\theta \qquad (3.37)$$

Substituting Equation (3.25) into equation (3.37) we get,

$$V\left(\frac{q}{L}\sin\theta\right) + A\frac{\partial V}{\partial t} + Q\frac{\partial V}{\partial x} + gA\frac{\partial y}{\partial x} + gA\left(S_f - S_0\right) = \frac{q}{L}\sin\theta \ u\cos\theta \qquad (3.38)$$

Noting that $Q = AV$ and dividing equation (3.38) throughout by A we get

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} + g\left(S_f - S_0\right) + \frac{V}{A}\left(\frac{q}{L}\sin\theta\right) = \frac{q}{A\,L}\sin\theta \ u\cos\theta \qquad (3.39)$$

Now rearranging the equation (3.39) we get,

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} + g\left(S_f - S_0\right) = \frac{q}{A\,L}\sin\theta \ \left(u\cos\theta - V\right) \qquad (3.40)$$

Equation (3.40) is the general momentum equation of an open channel with lateral inflow channel at varying angles.

3.5 Method of solution

In this study, we analyzed analytically how the variation of the angle in the lateral inflow channel affects discharge and depth in the main rectangular channel. The equations governing the flow considered in the problem are non-linear. The non-linearity is due to the term $V\frac{\partial V}{\partial x}$ in the momentum equation. These equations are non-linear first order partial differential equations. It is not possible to solve these equations using an exact method; thus, the finite difference method is used to obtain approximate solutions. In this method, the partial differential equations are estimated from a set of linear equations linking the values of the functions at each mesh point. Finally, these sets of algebraic equations are solved.

Accuracy is a measure of how well the discrete solution represents the exact solution of the problem. In other words, a technique is accurate if the truncation error is negligible. A technique is consistent if the truncation error decreases as the step size is reduced. A technique is stable if the errors in the solution will remain bounded (if the solution tends to infinity the method is unstable). Finally, a numerical technique is convergent if the solution approaches the exact solution as the grid spacing is reduced to zero. All these four factors have been proven in the finite difference method, Nicholas (1996). Hence, that is why it is used to solve the non-linear Saint Venant equations.

3.5.1 Finite difference method

The finite difference approximations of these partial differential equations are obtained from Taylor's series expansion of the independent variables. From definition

$$u_x = \frac{\partial \mathrm{u}}{\partial \mathrm{x}} = \lim_{\Delta \mathrm{x} \to 0} \frac{\mathrm{u}(\mathrm{x} + \Delta \mathrm{x}) - \mathrm{u}(\mathrm{x})}{\Delta \mathrm{x}} \tag{3.41}$$

This formula can be used as an approximation to the derivative of u at x taking Δx is very small. Now from Taylors series

$$\mathrm{u}(\mathrm{x} + \Delta \mathrm{x}) = \mathrm{u}(\mathrm{x}) + \Delta \mathrm{x}\, \mathrm{u}_{\mathrm{x}}(\mathrm{x}) + \frac{\Delta \mathrm{x}^2}{2} \mathrm{u}_{\mathrm{xx}}(\mathrm{x}) + \cdots \tag{3.42}$$

On rearranging we get

$$\frac{\mathrm{u}(\mathrm{x} + \Delta \mathrm{x}) - \mathrm{u}(\mathrm{x})}{\Delta \mathrm{x}} = \mathrm{u}_{\mathrm{x}}(\mathrm{x}) + \frac{\Delta \mathrm{x}}{2} \mathrm{u}_{\mathrm{xx}}(\mathrm{x}) + \cdots \tag{3.43}$$

If we consider the step Δx to be very small, then the square, the cube and higher powers of this step will be very small and hence the product of the step size and their derivatives will be negligible. From this fact the equation (3.43) truncates and reduces to

$$\mathrm{u}_{\mathrm{x}} = \frac{\mathrm{u}(\mathrm{x} + \Delta \mathrm{x}) - \mathrm{u}(\mathrm{x})}{\Delta \mathrm{x}} + 0(\Delta \mathrm{x}) \tag{3.44}$$

This equation is a first order finite approximation and it helps us to solve the non-linear Saint Venant equations. The above analysis deals with continuous solution, however the objective is to calculate *u* at a set of discrete points on the mesh, and this is the numerical solution. The numerical solution of equations (3.31) and (3.40) will be approximated from a rectangular grid approximated at a discrete number of points. This rectangular grid is obtained by dividing the (x, t) plane into a network of rectangles of sides Δx and Δt by drawing the set of lines where h and k are the equal spacing in the x an t axis respectively.

$x = i\Delta x = ih, \quad i = 0,1,2,\ldots$ (3.45)

$t = j\Delta t = jk, \quad j = o,1,2,\ldots$ (3.46)

The nodes or mesh points of the network occur at the intersections of the straight lines drawn parallel to the *x* and *t* axes. The lines parallel to the x axis represent time while those drawn parallel to the y axis represent locations along the channel. The location lines are drawn with spacing Δx while the time lines are drawn with spacing Δt.Two indices identify each node in the network. The first designates spatial point (location) of the node in the time while the second designates the time.

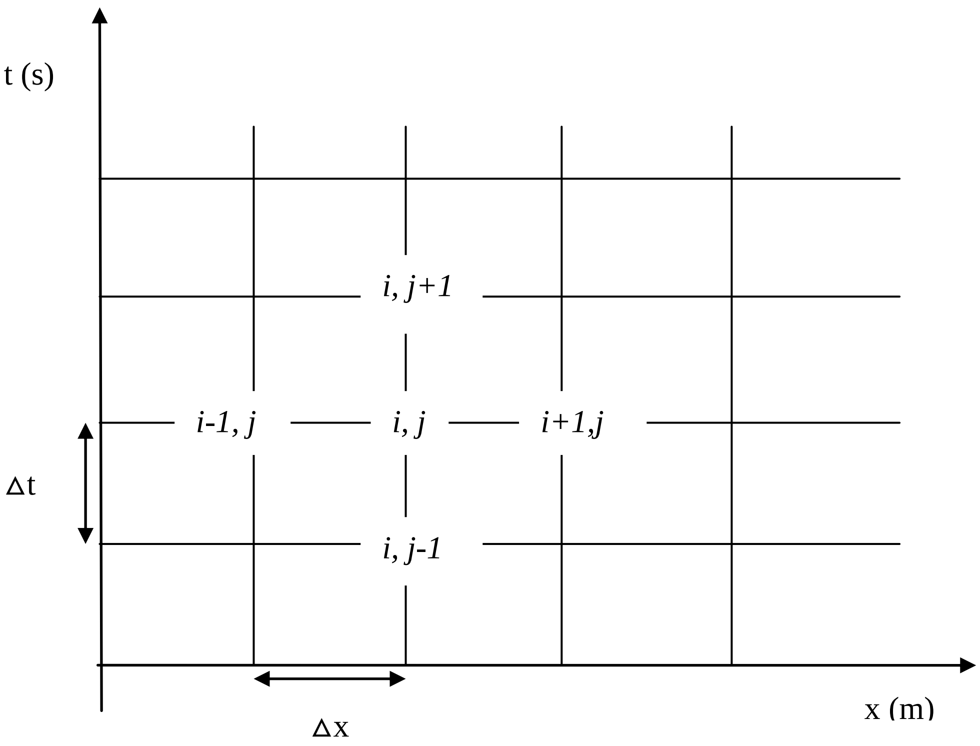

Figure 2: Finite difference mesh

If we let $u_{i,j}$ be the numerical approximation for $u(x_i\ ,t_j)$, then the first order forward difference approximation of the derivatives of V and y with respect to time t respectively are given by:

$$V_t = \frac{V_{i,j+1} - V_{i,j}}{k} + 0(k) \qquad (3.47)$$

$$y_t = \frac{y_{i,j+1} - y_{i,j}}{k} + \ 0(k) \qquad (3.48)$$

Similarly the first order finite difference derivatives of V and y with respect to x respectively are given by:

$$V_x = \frac{V_{i+1,j} - V_{i,j}}{h} + 0(h) \qquad (3.49)$$

$$y_x = \frac{y_{i+1,j} - y_{i,j}}{h} + 0(h) \qquad (3.50)$$

Now the governing equations (3.31) and (3.40) are replaced by the finite difference analogies of the partial differential equations.

3.6 Governing equations in finite difference form

The equations (3.31) and (3.40) are non-linear hence cannot be solved analytically hence we have to establish the finite difference method to solve them subject to the initial conditions

$$V(x,0) = \ 0, \quad y(x,0) = 0 \qquad for\ all\ x > 0 \qquad (3.51)$$

boundary conditions

$$V(0,t) = \ V_0, \quad y(0,t) = y_0 \qquad for\ all\ t > 0 \qquad (3.52)$$

$$V(x_N, \text{t}) = \ V_0, \quad y(x_N,t) = y_0 \qquad for\ all\ t > 0 \qquad (3.53)$$

The point x_N refers to the exit point of the section of the open rectangular channel. the considered section of the channel was 10m.Because of numerical unstable solutions of the implicit finite difference method, Viessman et al (1972) noted that more stable solutions could be obtained by diffusing the finite difference approximations.

$$\frac{\partial V}{\partial t} = \frac{V(i, j+1) - 0.5\big(V(i-1, j) + V(i+1, j)\big)}{\triangle t} \tag{3.54}$$

$$\frac{\partial y}{\partial t} = \frac{y(i, j+1) - 0.5\big(y(i-1, j) + y(i+1, j)\big)}{\triangle t} \tag{3.55}$$

$$S_f = \frac{S_f(i-1, j) + S_f(i+1, j)}{2} \tag{3.56}$$

$$\frac{\partial V}{\partial x} = \frac{V(i+1, j) - V(i-1, j)}{2 \triangle x} \tag{3.57}$$

$$\frac{\partial y}{\partial x} = \frac{y(i+1, j) - y(i-1, j)}{2 \triangle x} \tag{3.58}$$

Now, we convert equation (3.3) into finite difference form. Therefore, we have,

$$\frac{y(i, j+1) - 0.5\big(y(i-1, j) + y(i+1, j)\big)}{\triangle t} + V(i, j)\frac{y(i+1, j) - y(i-1, j)}{2 \triangle x} + \frac{A}{T}\frac{V(i+1, j) - V(i-1, j)}{2 \triangle x} = \frac{q}{T\,L}\sin\theta \tag{3.59}$$

$$y(i, j+1) = 0.5\big(y(i-1, j) + y(i+1, j)\big) - \triangle t\left\{V(i, j)\frac{y(i+1, j) - y(i-1, j)}{2 \triangle x} + \frac{A}{T}\frac{V(i+1, j) - V(i-1, j)}{2 \triangle x} - \frac{q}{T\,L}\sin\theta\right\} \tag{3.60}$$

Equation (3.60) is the finite difference method for the continuity equation for an open channel with lateral intake at an angle.Now we convert the momentum equation (3.40) into the finite difference form. We have,

$$\frac{V(i,j+1) - 0.5\big(V(i-1,j) + V(i+1,j)\big)}{\triangle t} + V(i,j)\frac{V(i+1,j) - V(i-1,j)}{2 \triangle x}$$
$$+g\frac{y(i+1,j) - y(i-1,j)}{2 \triangle x} + g\left(\frac{S_f(i-1,j) + S_f(i+1,j)}{2} - S_0\right)$$
$$= \frac{q}{A\,L}\sin\theta\ \left(u\cos\theta - V(i,j)\right) \tag{3.61}$$

The friction slope S_f in unsteady flow can be estimated by either using the Chezy or the Manning resistance equations. In this work we decide to use the Manning resistance equation. From the Manning equation,

$$V = \frac{1}{n}R^{\frac{2}{3}}S_f^{\frac{1}{2}} \tag{3.62}$$

Now letting S_f to be our friction slope and making it the subject of the formula we have

$$S_f^{\frac{1}{2}} = \frac{V\,n}{R^{\frac{2}{3}}} \tag{3.63}$$

Now squaring both sides, we get

$$S_f = \frac{V^2n^2}{R^{\frac{4}{3}}} \tag{3.64}$$

Substituting equation (3.64) into equation (3.61) we get

$$V(i,j+1) = 0.5\big(V(i-1,j) + V(i+1,j)\big) - \triangle t\left\{V(i,j)\frac{V(i+1,j) - V(i-1,j)}{2 \triangle x}\right.$$
$$+g\frac{y(i+1,j) - y(i-1,j)}{2 \triangle x} + g\left[\frac{n^2}{2R^{\frac{4}{3}}}\big(V^2(i-1,j) + V^2(i+1,j)\big) - S_0\right]$$
$$\left. - \frac{q}{A\,L}\sin\theta\ \left(u\cos\theta - V(i,j)\right)\right\} \tag{3.65}$$

The equations (3.60) and (3.65) are the continuity and the momentum equations respectively in finite difference form of an open channel with lateral intake at an angle. The index i refer to spatial points, whereas the index j refers to time. The terms $y(i, j+1)$ and $V(i, j+1)$ in equations (3.60) and (3.65) respectively, are computed subject to the initial and boundary conditions below.

Now taking the velocity V_o =10 m/s and depth of the channel to be y_o = 0.5 m, the initial and boundary conditions in finite difference form become where the index i stands for x which is the distance along the channel while j stands for time.

Initial conditions,

$$V(i,0) = 0 \qquad y(i,0) = 0 \tag{3.66}$$

The boundary conditions

$$V(0,j) = 10 \qquad y(0,j) = 10 \tag{3.67}$$

$$V(N,j) = 10 \qquad y(N,j) = 10 \tag{3.68}$$

The two equations are solved using very small values of $\triangle t$. In this research, we set $\triangle x = 0.1$ and $\triangle t = 0.0001$. This finite difference method is known to be convergent and numerically stable whenever $\frac{\triangle t}{(\triangle x)^2} < \frac{1}{2}$ .The number of sub-divisions along the channel was taken to be 100 while along the time was taken to be 10000 sub-divisions.

CHAPTER FOUR

RESULTS AND DISCUSSION

The Matlab software is used to simulate the equations (3.60) and (3.65) which appear in Appendix 1. This was done by varying i and j at various nodal points. Then various graphs were plotted using the values of the velocity V and the depth y at a certain location. Various flow parameters of width, slope, roughness coefficient, energy coefficient at an angle zero were investigated to determine how they affect the velocity. This was done so as to compare with the results obtained by Kwanza *et al* (2007). With the introduction of the lateral inflow channel, various flow parameters of angle, the cross-sectional area, velocity and length of the lateral inflow channel were varied to investigate how they affect the velocity in the main open channel. The following graphs for different values of the flow parameters mentioned above.

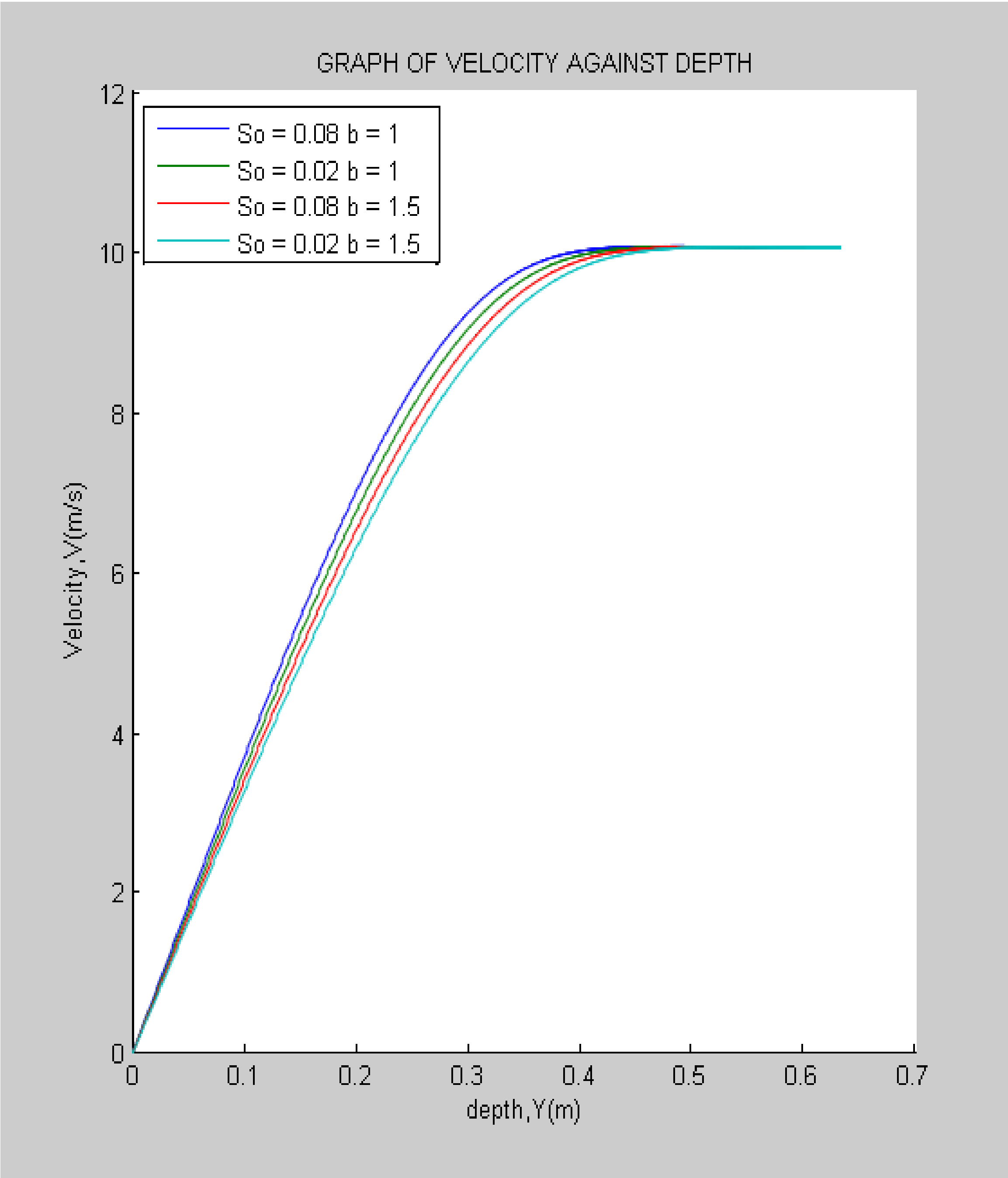

Figure 3: Velocity profiles versus depth at angle zero for varying width and slope of the channel

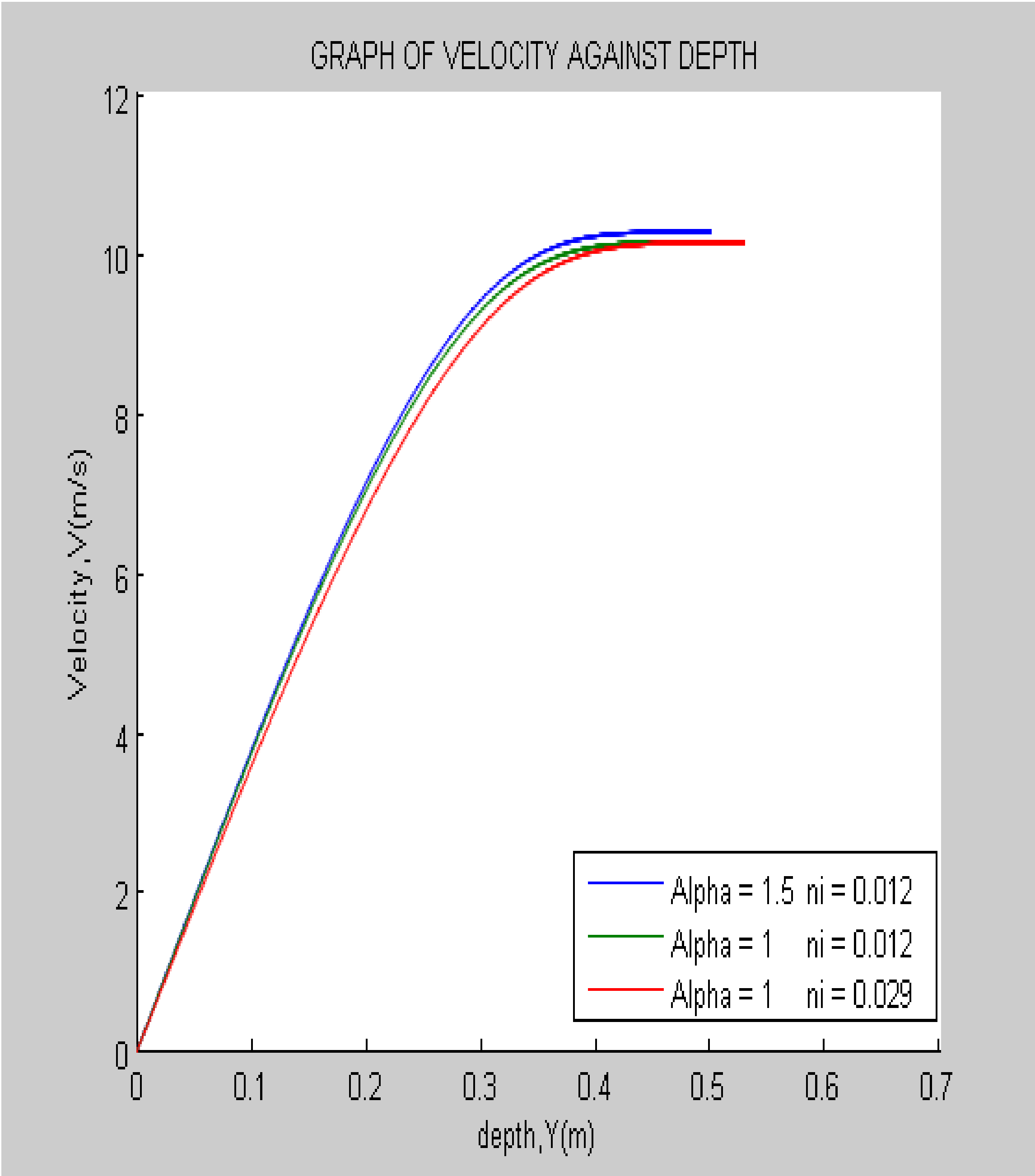

Figure 4: Velocity profiles versus depth at angle zero for varying roughness and energy coefficients

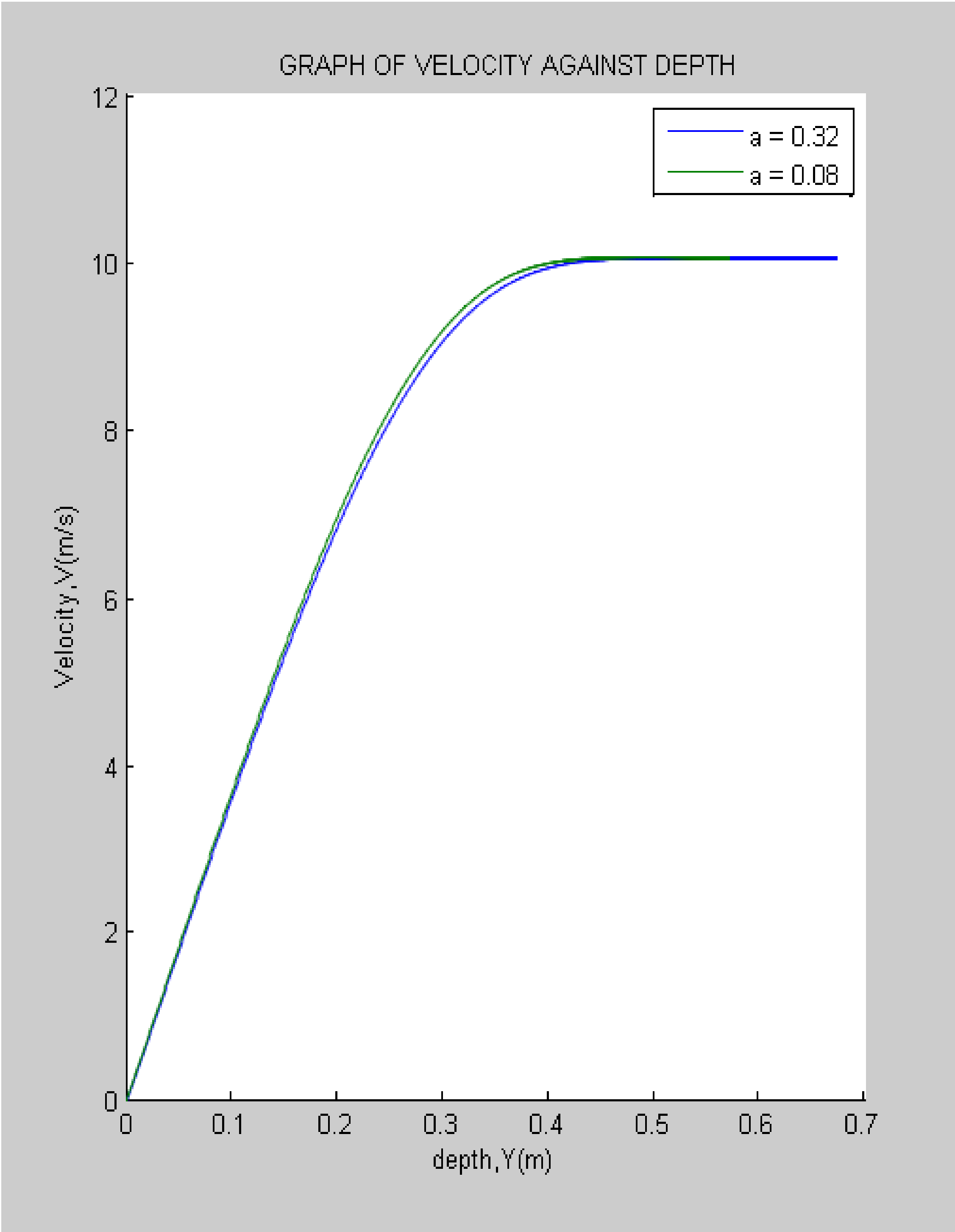

Figure 5: Velocity profiles versus depth at varying cross-sectional area of the lateral inflow channel at angle 40^0.

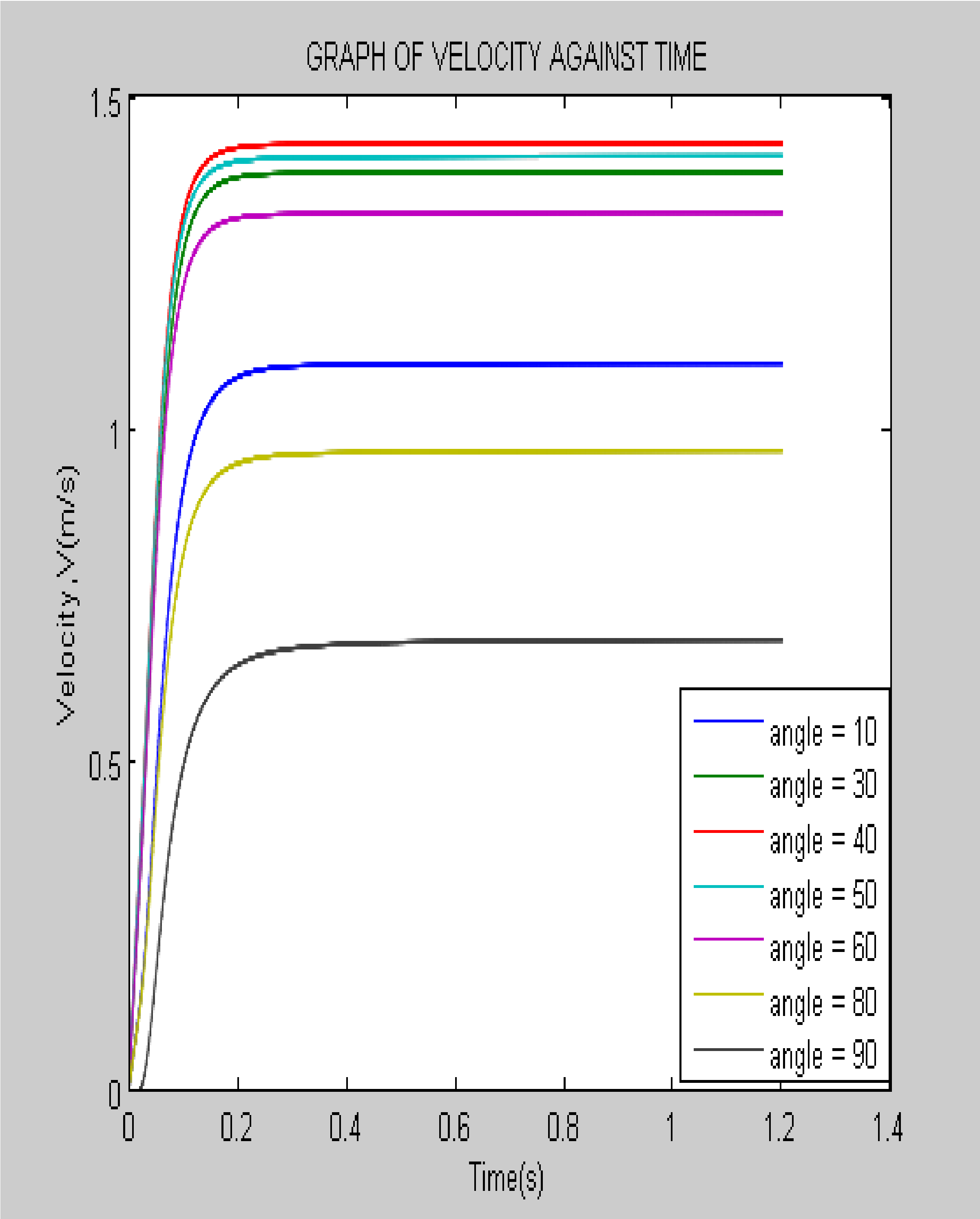

Figure 6: Velocity profiles versus time along the channel for varying angle of lateral inflow channel**.**

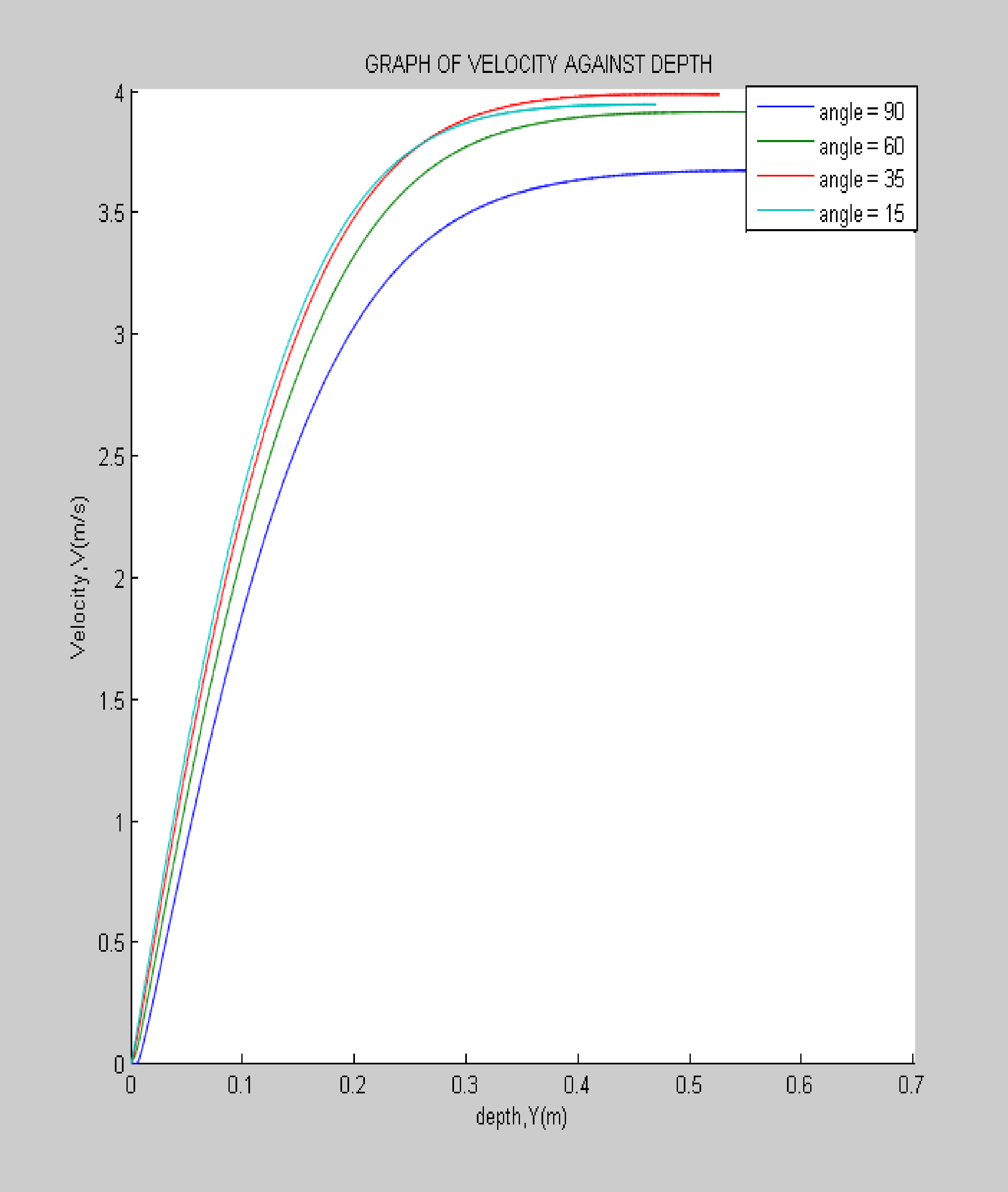

Figure 7: Velocity profiles versus depth for varying angle of the lateral inflow channel

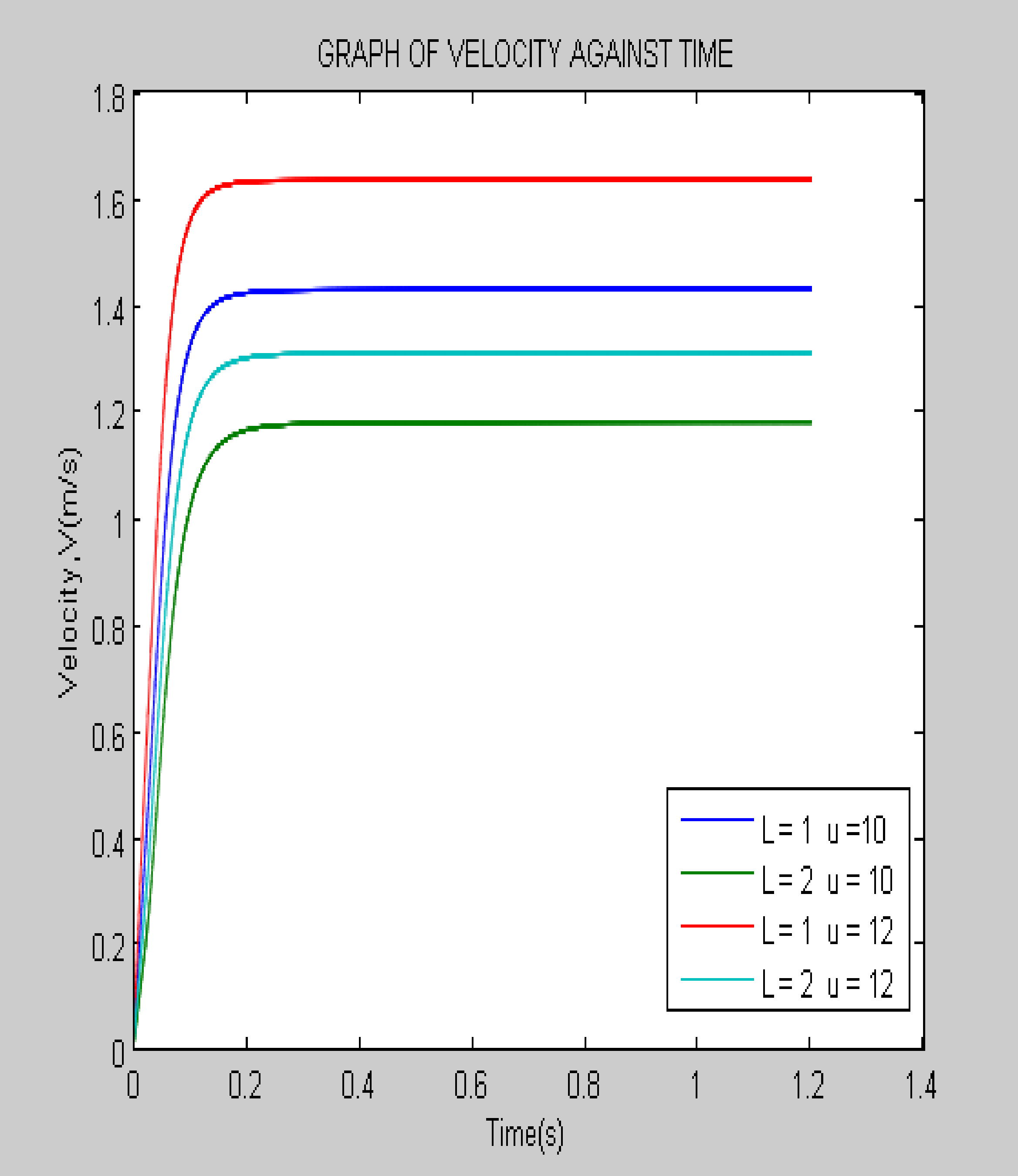

Figure 8: Velocity profiles versus time along the channel for varying length and velocity of the lateral inflow channel at angle 40^0

Figure 3 and 4 correspond to the findings of an open rectangular channel with no lateral discharge; if the angle is zero, it means there is no lateral channel.

From figure 3, it is noted that an increase in the width of the channel from 1m to 2m leads to a decrease in the flow velocity. This is because an increase in the width results in the increase in the wetted perimeter of the fluid under the conduit. This increase in the wetted perimeter will lead to an increase in the shear stress in the channel bottom, which will result in a reduction in the flow velocity. It is also noted that an increase in the slope will lead to an increase in the fluid flow velocity. Thus, an increase in the slope from 0.002 to 0.08 will lead to the increase in the flow velocity. This can be seen in the Manning formula where an increase in the slope results in an increase in the flow velocity since the two are directly proportional to each other.

From figure 4, it is noted that a decrease in the roughness coefficient from 0.029 to 0.012 will lead to an increase in the flow velocity. This is because an increase in the roughness coefficient leads to increase in the shear stresses at the sides of the channel which reduces the velocity. This increase in the shear stress leads to a reduction of the motion of the fluid particles near or at the surface of the conduit. The speed of the particles which neighbor these slow moving molecules will be reduced. This will result in the overall velocity of the fluid being reduced. We also note that an increase in the energy coefficient, Alpha, from 1 to 2 will lead to an increase in the flow velocity of the fluid. An increase in the energy coefficient will lead to an increase in the fluid particle's energy. This increase in the fluid particle's energy results to the particles attaining more random motion which causes constant bombardments with other fluid particles which leads to an increase in the flow velocity of the particles. This increase in the velocity of the particles will lead to an increase in the velocity of the fluid.

From figure 5, an increase in the cross-sectional area from 0.08 m^2 to 0.32 m^2 of the lateral inflow channel leads to a decrease in the flow velocity of the main channel. An increase in the area will lead to an increase in the wetted perimeter of the lateral inflow channel because the fluid will spread more in along the conduit. A large wetted perimeter leads to large shear stress at the sides of the channel which results to the flow velocity being reduced. Moreover, an increase in depth will lead to an increase in the velocity. It is also noted that at a depth of about 0.45m that is where maximum velocity occurs. The fluid flow velocity at the channel bottom is zero due to the non-slip condition of fluids. The non-slip condition states that a fluid in contact with a surface will achieve the same velocity at the surface. Since at the channel bottom the surface is not moving, the flow velocity at this section of the channel will be zero. However, as you move vertically upwards, the velocity increases since the frictional forces decrease and velocity becomes maximum slightly below the free surface. At the free surface the velocity is not maximum due to surface tension that is caused by strong cohesive forces between the fluid molecules. Moreover, the wind blowing across the free surface causes frictional forces which reduce the fluid flow velocity. The velocity is much lower when the wind is blowing in the opposite direction.

From figure 6, an increase in the angle of the lateral inflow channel does not necessarily mean an increase in the fluid flow velocity of the open rectangular channel. We see that an angle of 40^0 has a higher velocity value than an angle of 90^0. Those angles of 10^0, 60^0, 80^0 and 90^0 have lower values of velocity compared to 30^0, 40^0 and 50^0 angles. Moreover, figure 7 also shows that at 35^0 of the lateral inflow channel has higher value of velocity compared to other angles. Since the discharge is directly proportional to velocity, for one to get a maximum discharge from the lateral inflow channel, one has to construct it with an angle ranging from 30^0 to 50^0. The reason why the flow velocity in the open channel at an angle of 90^0 of the lateral inflow channel is lower is because the velocity in this lateral inflow channel is maximized. At this angle, turbulence occurs in the section where the two channels meet resulting in a reduction in the flow velocity in the main channel.

From figure 8, it is observed that an increase in the velocity of the lateral inflow channel from 10m/s to 12 m/s will lead to an increase in the flow velocity of the main open channel. However, an increase in the length of the lateral inflow channel from 1m to 2m leads to a reduction in the flow velocity in the main open channel. An increase in the flow velocity of the lateral inflow channel means that more fluid particles at a given time will collide with the fluid particles in the main open channel resulting in more random motion of the particles. This random motion leads to bombardments between the fluid particles leading to an increase in the velocity of the particles which in turn will lead to an increase in the fluid flow velocity. An increase in the length of the lateral inflow channel will lead to a decrease in the velocity. This is due to the increase in the shear stress on the walls and the channel bottom.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

Conclusion

The objectives of modeling fluid flow in an open channel with a lateral inflow channel have been achieved. The results also compare with earlier research done when various flow parameters are compared. Moreover, investigations of the effects of varying angle, the cross-sectional area, velocity and length of the lateral inflow channel on velocity in the main open in the summary are that,

1. Increasing the angle of the lateral inflow channel does not necessarily mean an increase in the velocity in the main open channel. That angles of between 30^0 and 50^0 exhibits higher values of velocity in the main open channel than other angles.
2. Increasing the cross-sectional area of the lateral inflow channel leads to decrease in the flow velocity in the main open channel.
3. Increasing the velocity in the lateral inflow channel leads to an increase in the flow velocity in the main open channel.
4. Increasing the length of the lateral inflow channel leads to a decrease in the velocity in the main open channel.

Recommendations

There is still a need to compare experimentally the theoretical results found in this research with laboratory results. Rectangular channels with lateral inflow channels can be developed in a laboratory and investigations on how the various flow parameters like length of the lateral inflow channel affects the discharge in the open channel. Finally, it is recommended that future research should be carried out on

- The effect of lateral outflow channel on discharge

- The effect of two or more lateral inflow channels at various locations on discharge in the main channel
- Flow in a trapezoidal, triangular or circular open channel with lateral inflow/outflow channel
- Solving the above problem using a different numerical technique like perturbation or finite element method

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APPENDIX 1

Matlab code

The finite difference equations (3.10) and (3.65) were solved simultaneously using the Matlab code below and subject to the initial (3.66) and the boundary conditions (3.67) and (3.68). The various results were obtained by varying the flow parameters of cross-sectional area of flow, velocity of flow, length of flow and angle of the lateral inflow channel on how each affects the velocity of the main open rectangular channel.

```
function lateralintakechannel()
%generate array x value
N=100;
x1=0;
xN=10;
dx =(xN-x1)/N;
x = zeros(1,N);

%generate array t values
K=10000;
t1=0;
tK=1;
dt=(tK-t1)/K;
t=zeros(1,K);

Y= zeros(N,K);
V= zeros(N,K);

%constants
g=9.81; So=0.002; yi=0.5; ni=0.012; b=1; Alpha=1;
b1=0.6; y2=0.3; u=10; L=1;
```

```
a=b1*y2;
A=b*yi;
P=b+(2*yi);

%variable angle
angle=35;
theta = (angle*pi/180);

for k=1:K+1
        V(1,k)=10;  Y(1,k)=0.5;                 %ENTRY
        V(N,k)=10;  Y(N,k)=0.5;                 %EXIT
        t(k)=(k-1)*dt;
 end
for i=1:N+1
        V(i,1)=0;  Y(i,1)=0;
        x(i)=(i-1)*dx;
end
for k=1:K
for i=2:N
    Y(i,k+1)= 0.5*(Y(i-1,k)+Y(i+1,k))-dt*((A/b)*(V(i+1,k)-V(i-
1,k))/(2*dx)-V(i,k)*(Y(i+1,k)-Y(i-1,k))/(2*dx)-(a*u*sin(theta))/(b*L));

    V(i,k+1)= 0.5*(V(i-1,k)+V(i+1,k))-dt*(Alpha*V(i,k)*(V(i+1,k)-V(i-
1,k))/(2*dx)+g*(Y(i+1,k)-Y(i-1,k))/(2*dx)-g*(So-
((0.5*(ni^2)/(A/P)^(4/3))*(V(i-1,k)^2)+(V(i+1,k)^2))))+
dt*a*u*sin(theta)*((u*cos(theta))-(V(i,k)))/(L*A);
end
end
```

```
% figure
%   hold all;
%         plot(Y(7,:),V(7,:));
%         title('GRAPH OF VELOCITY AGAINST DEPTH');
%         xlabel('depth,Y(m)');
%         ylabel('Velocity,V(m/s)');
%         [~,~,~,current_entries] =legend;
%         legend([current_entries {sprintf([texlabel('a'),' = %g'],a)}]);
```

APPENDIX 2

Publication

Macharia Karimi, David Theuri and Mathew Kinyanjui. Modelling Fluid Flow in an Open Rectangular Channel with Lateral Inflow Channel.

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