Effects of Variable Viscosity on Unsteady Natural Convection Hydromagnetic Flow past an Isothermal Sphere

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DECLARATION

This Thesis is my original work and has not been presented for a degree award in any other university.

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This Thesis has been submitted for examination with our approval as university supervisors.

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DEDICATION

This thesis is dedicated to my family and friends. A special feeling of gratitude to my parents, Mr. Ndirangu Mwangi and Mrs. Rahab Njeri Mwangi for being there for me. Am grateful Uncle Joseph Kimama Mwangi and Aunt Grace Wambui Mwangi for the relentless support you have given towards my study. To my best friend Stephen Kamau, I thank you for all the support you have given towards my research.

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NOMENCLATURE

β	Volumetric coefficient of thermal expansion,$[(C^0)^{-1}]$
δ_0	Electric conduction, $[S/m]$
$\hat{\bigtriangledown}$	Gradient operator $\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
$\hat{F}$	Body forces vector in x and y directions, $[N]$
μ	Viscosity of the fluid,$[Ns/m^2]$
ρ	Density of the fluid,$[kg/m^3]$
τ_w	Shearing stress,$[N/m^2]$
θ	Dimensionless temperature function
ϑ	Reference Kinematic Viscosity, $[m^2/k]$
a	Radius of the sphere,$[m]$
B_0	Strength of magnetic field, $[A/m]$
C_f	Skin friction coefficient,$[Ns^2/kg]$
C_p	Specific heat at constant pressure, $[J\,\text{deg}^{-1}\,\text{kg}^{-1}]$
C_{fx}	Local skin friction coefficient, $[Ns^2/kg]$
f	Dimensionless stream function
g	Acceleration due to gravity, $[m^2/s]$
Gr	Grashof number
h_m	Magnetic field intensity, $[T]$
K	Thermal conductivity of the fluid,$[W/mK]$

M	Magnetic Parameter
Nu	Nusselt number
o	The center of the sphere
Pr	Prandtl number
Q	Heat generation parameter
q_w	Heat flux at the surface,$[W/m^2]$
$r(\hat{x})$	Radial distance from the symmetrical axis to the surface of the sphere, $[m]$
T	Temperature of the fluid, $[K]$
T^*	Dimensional Temperature$[K]$
t^*	Dimensional Time,$[S]$
T_∞	Temperature of the ambient fluid,$[K]$
T_w	Temperature at the surface,$[K]$
u, v	Dimensionless velocity component
u^*, v^*, w^*	Dimensional Velocity component$[m/s]$
V	Fluid velocity in the x,y direction, $[m/s]$
x, y	Axis direction
x^*, y^*, z^*	Dimensional Axis direction$[M]$

ABBREVIATIONS

DNS	Direct Numerical Scheme
PDEs	Partial Differential Equations
MATLAB	Matrix Laboratory
FDMs	Finite Difference Methods

ABSTRACT

In this study, the effects of variable viscosity on unsteady natural convection hydromagnetic flow of an incompressible electrically conducting fluid, past an isothermal sphere was investigated. The non-linear partial differential equations governing the flow were solved numerically using the finite differences method and implemented using MATLAB software. The velocity profiles, the temperature profiles, the skin friction, heat transfer and the effect of varying the various flow parameters are discussed. These flow parameters include the Grashof number,Reynolds number, the magnetic parameter and the viscous variation parameter. A change on the parameters is observed to either increase, decrease or to have no effect on the flow.

Chapter 1

INTRODUCTION

1.1 Background

A Fluid is a substance that undergoes through a process of continuous deformation when subjected to a certain amount of shearing stress. It can also be referred to as a state such as liquid or gas in which the molecules can move past one another. A fluid can also be described as a substance that has the capability to flow. Fluids flow easily and conform to the shape of their containers. A fluid may be a liquid, vapor or gas. A vapor denotes a gaseous substance interacting with its own liquid phase for example steam above water. Gases have weak intermolecular forces and and expand to fill any container. When gases are left free, they expand and form the atmosphere of the earth. Gases are highly compressible. Liquids have strong intermolecular forces and tend to retain constant volume. Placed in a container, a liquid occupies only its own volume and form a free surface which is at the same pressure as any gas touching it. Liquids are nearly incompressible.

There are various classification of fluids which include: Ideal fluids and Real Fluids. An ideal fluid is an imaginary fluid which has no viscosity, its incompressible and has no surface tension. Ideal fluids do not exist in the real world. Real fluids are the ones that have viscosity, surface tension and compressibility in addition to the density. These types of fluids exists in the real world. Real fluids can also be classified as Newtonian and Non-newtonian fluids. Newtonian fluids are the simplest models of fluids that account for viscosity. This means that the viscosity of a Newtonian Fluid remain constant no matter the amount of shear applied at constant temperature. Examples of Newtonian fluids are water and air which can be assumed Newtonian for practical calculations under ordinary conditions. Non-Newtonian fluids are fluids where the viscosity of the fluid is dependent on the shear rate or shear rate history and their properties are totally different from those of Newtonian fluids Therefore, their viscosity or flow behaviour change under stress. Examples of Non-Newtonian fluids include a mixture of cornstarch and water, Ketchup, paints and blood.

Fluid flow is the motion of a fluid subjected to unbalanced forces or stresses. The motion continues as long as unbalanced forces are applied. Fluid flows can be described based on their dependence on time as either steady or unsteady. A steady flow is one in which all flow for example velocity, temperature, pressure and density does not depend on time while for unsteady flows, the flow variables are dependent on time. Other types of flows include laminar, Turbulent, Compressible, Incompressible, Uniform and Non-Uniform Fluid flows.

1.1.1 Ferrofluid

A ferrofluid is a liquid that becomes strongly magnetized in the presence of magnetic field. These fluids are used in forming liquid seals around the spinning drive shafts in hard disks, used in semi-active dampers in mechanical and aerospace applications, used to image magnetic domain structures on the surface of ferromagnetic materials and in magnetic drug targeting.

1.1.2 Fluidity

This refers to the quality or state of being a fluid. It can also be referred to as the physical property of a substance that enables it to flow.

1.1.3 Isothermal Sphere

An Isothermal sphere is a perfectly round geometrical object in three dimensional space analogous to the surface of a complete round ball in which heat is uniformly distributed at constant temperature. In this study, a sphere which is completely immersed in a ferrofluid and maintained at constant temperature is referred to as an Isothermal sphere.

1.1.4 Temperature Dependence of Fluid Viscosity

This refers to a phenomenon where as the temperature of the liquid increases, the liquid viscosity tends to decreases. It can also be referred to as a situation where the fluidity increases. For example; the cooking oil appears to move more fluidity upon frying pan after being heated by a stove.

1.1.5 Natural Convection Fluid Flow

This is a mechanism that is caused by the action of density gradient in conjunction with gravitational field. This is an important mechanism that operates in a variety of environments from cooling electronic circuit boards in computers to causing a large scale circulation in the atmosphere as well as in lakes and oceans that influences the weather. It can also be explained as a type of heat transport in which the fluid motion is not generated by any external source but only by density differences in the fluid occurring due to temperature gradients.

Convection occurs when particles with a lot of heat energy in a liquid or gas move and take the place of particles with less heat energy. Natural convection occurs by natural means such as buoyancy. Since the fluid velocity associated with natural convection is relatively low, the heat transfer coefficient encounters in natural convection is also low.

1.1.6 Hydrodynamics

Hydrodynamics is a branch of fluid mechanics which deal with the study of motion of incompressible liquids which is influenced by internal and external forces. It can also be defined as a branch of physics that deals with the motion of fluids and the forces acting on solid bodies immersed in fluids. This study is mainly applied in engineering.

Examples of other applications include: determining the mass flow rate of petroleum through pipelines, measuring flows around bridge pylons and off shore rigs, ship hull designing, optimizing propulsion efficiency, predicting weather patterns and wave dynamics and measuring liquid metal flows. Hydrodynamics also deals with the analysis of decreased fuel consumption, reduced drag on structures, minimizing noise and vibration and mitigating unwanted effects, like fouling.

1.1.7 Dimensional Analysis

Dimensional analysis is the practice of checking relations among physical quantities by identifying their dimensions. The dimension of any physical quantity is the combination of the basic physical dimensions that compose it. Some fundamental physical dimensions are length, mass and time. All other physical quantities can be expressed in terms of these fundamental physical dimensions. It offers a method for reducing complex physical problems to the simplest form prior to obtaining a quantitative answer. It's based on the principle of dimensional homogeneity which states that every term in an equation when reduced to fundamental dimensions must contain identical powers of each dimension. Each physical quantity can be expressed by an equation giving a relationship between different dimensional and non-dimensional quantities. Therefore, dimensional analysis helps in determining a systematic arrangement of variables in the physical relationship combining dimensional variables to form non-dimensional parameters.

1.2 Problem statement

Figure 1.1 below represents a Uniformly heated sphere of radius(a)and center (o)which is immersed in a two- dimensional laminar free convective fluid flow. The fluid is considered to be viscous and incompressible. Viscosity of the fluid is taken as a linear function of temperature which implies that viscosity varies directly proportional to temperature. In this case, the fluid flow is considered to be unsteady which implies that velocity of the fluid varies with time.

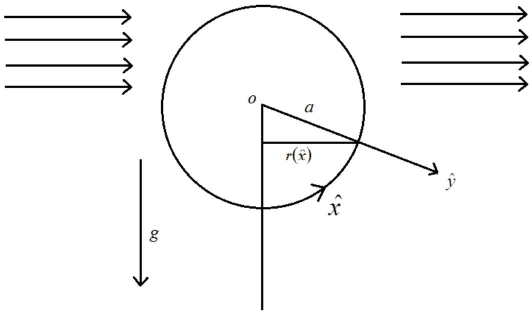

Figure 1.1: Physical Model of the Problem

1.3 Justification of the study

This study finds its application in various places such as industries, Technology and in Medical Fields. In industries, this study can be applied in the manufacture of motorbike wheels, seals, loudspeakers, dampers and electric motors. In medical fields, this study finds application in pathogen detection, hyperthermia and MRI contrast agent. In technological field, this study finds application in the Rotary Feedthrough and Diesel particulate Trap generation.

1.4 Objectives

1.4.1 General Objective

To analyze the effects of variable viscosity on unsteady natural convection hydromagnetic fluid flow past an isothermal sphere.

1.4.2 Specific Objectives

i. To determine the effects of magnetic field, Grashof number, Reynolds number and viscosity on the fluid velocity, temperature,rate of skin friction on the surface of the sphere and rate of heat transfer on the surface of the sphere.

In the next chapter, literature review of the previous studies done in this field have been considered. This review considers highlighting the work done in the same field by other scholars.

Chapter 2

LITERATURE REVIEW

The study of electric conducting fluid flows has gained popularity in our world today. These fluids include plasmas, liquid metals, salt water and air. This has attracted many researchers to carry out research in the same field. This is because it has found its application in many areas since it involves study of electrically conducting fluids. These fluids can be found in areas such as electrical and geothermal power plants in Kenya. The interaction of the current with the magnetic field changes the motion of the fluid and produces an induced magnetic field. An isothermal sphere in this study is a uniformly heated perfect geometrical object that is made of non-conducting material. Abd-el Malek et al. (2004) investigated similarity solutions for unsteady free-convection from a continuous moving vertical surface. These researchers used the shooting method to solve the obtained differential equations to obtain analytical solution for temperature and numerically for velocity. Their results showed that increase in Prandtl number Pr leads to decrease in the thickness of the thermal boundary-layer and also a decrease in the vertical velocity u.

Natural convection flow from an isothermal horizontal cylinder was investigated by Molla et al. (2005) where viscosity was taken as an inverse function of temperature for fluids having large Prandtl number. The researchers observed that there was an increase in the rate of heat transfer and a decrease in the skin-friction coefficient due to the effect of Rayleigh number (Ra) and viscosity variation parameter. It was also observed that the momentum and thermal boundary layer become thinner where there is an increase in the values of viscosity-variation parameter. Viscosity and velocity distribution were observed to increase whereas the temperature distributions were observed to decrease with the effect of Rayleigh number Ra and there was an enhancement of the thickness

of momentum boundary layer.
Unsteady natural convection flow past a vertical accelerated plate has been investigated by Deka and Neog (2009) where the plate was placed in a thermally stratified fluid. In their study, they used the Laplace transform techniques to solve the equations that they obtained. They observed that thermal stratification parameter S is an important factor in the study of unsteady vertical natural convection flow. This is due to the fact that velocity in thermally stratified fluid decreases with the increase of stratification parameter S and increases when Gr and t increase. They also observed that there is a decrease in temperature with increase in the value of Grashof number and Stratification parameter S. It was observed that Skin Friction, Nusselt number increases with increase in the values of S.
Unsteady magnetohydrodynamic heat transfer with thermal radiation flux in a semi-infinite porous medium was investigated by Bég et al. (2011). In this case, they considered analytical and numerical study. They observed that an increase in Hartmann number through a strong magnetic flux density causes a decrease in the flow velocity, u with distance normal to the plate surface into the boundary layer. This shows that changes in Hartmann number cause a significant change in the velocity. Their results showed that thermal radiation increase fluid temperatures and accelerates the flow whereas magnetic field simulated by the Hartmann number decelerates the flow and reduces the shear stress. They also observed that Darcian drug impedes the flow and increasing free convection accelerates the flow due to the effects of bouyancy forces. It was concluded that increase in Prandtl number decreases temperature whereas the velocity gradient increases.
The unsteady free convection flow with thermal radiation past a porous vertical plate with Newtonian heating was studied by Mebine and Adigio (2011). They used the technique of Laplace transform in deriving the solutions to the governing equations. It was concluded that increase in suction, radiation parameter, blowing and free convection parameter leads to increase in the velocity where it reaches to a maximum point and then starts to decrease up to zero at the edge of the boundary layer. They also observed

that there is a decrease in temperature as radiation parameter and suction increases whereas increase in blowing retards the flux of heat to the flow and there is an increase in temperature due to increase in suction.

Ramesh et al. (2011) investigated unsteady flow of a conducting dusty fluid between two circular cylinder. The variable separable method was used in their study in order to obtain the solutions. In their results, they observed that the graphs of velocity profiles are parabolic in nature for different values of Hartmann number and Time (T). Another observation by these researchers is that magnetic field retards the flow of both the fluid and dust phases as shown by the effects of Hartmann number and time increase leads to decrease in velocity. When the dust particles are very fine, then the velocities for both fluid and dust are the same.

Effect of inclined magnetic field on unsteady free convection flow of a dusty and viscous fluid between two infinite flat plates filled by a porous medium was investigated by Sandeep and Sugunamma (2013). They used the perturbative technique to solve the governing equations. In their results, they observed that when $Gr > 0$, velocity decreases with increase of magnetic parameter and also when (Porous parameter) increases.

It was observed that there is gradual decrease of velocity with increase in time and Heat source parameter. It was also observed that increase in Gr causes gradual increase in fluid velocity and particle velocity and that as Pr increases, the velocity of both the fluid and the particle phase decreases. The researchers also found out that an increase in (Volume fraction of dusty particles) causes an increase in fluid velocity and particle velocity and that there is a decrease in temperature of the dusty fluid when there is an increase in time.

Mutua et al. (2013) investigated the magnetohydromagnetic free convection flow of a heat generating fluid past a semi-infinite vertical porous plate. In this case, they considered the plate with variable suction. They used the finite difference method and observed that decrease in rotational parameter leads to increase in the velocity profiles. It was also observed that there was an increase in the velocity profiles when Eckert number and Magnetic parameter increase and on removal of injection. There was no effect observed on the primary velocity profile when Suction parameter was increased but there was a decrease in the secondary velocity profile.

Ramana (2013) studied the MHD free convective mass transfer flow past an infinite vertical porous plate with variable suction. They also considered the study with soret effects. In their study, they observed that increase in Prandtl number leads to decrease in velocity and increase in Sorect number also leads to increase in the velocity field. Also decrease in the applied magnetic intensity contributes to a decrease in velocity whereas a decrease in temperature was observed when Prandtl number increases.

Recently, Deepa and Murali (2014) investigated the effects of viscous dissipation on unsteady MHD free convective flow. They considered the convective flow with thermophoresis past a radiate, inclined and permeable plate. The implicit finite difference scheme with shooting method was used to solve the obtained governing equations. They found out that there is an increase in the viscous drag as well as the rate of heat transfer when there is a significant variation of viscosity. They made a conclusion that there is an induced concentration of the particles for a destructive reaction and reduction of generative reaction when there is higher order of chemical reaction.

The combined effect of variable viscosity and thermal conductivity of a viscous fluid on free convection flow in a vertical channel using DTM has also been studied by Umavathi and Shekar (2016). It was observed that increasing the viscosity led to enhancement of the flow and heat transfer. Also, increasing the variable thermal conductivity led to suppression of the flow and the heat transfer for variable viscosity. Unsteady natural hydrodynamic convection flow is a phenomenon that is of great importance in our world today.

Mwangi and Surindar (2016) studied the effects of temperature dependent viscosity on magnetohdrodynamic natural convection flow past an isothermal sphere and observed that increase in Magnetic parameter leads to decrease in velocity and temperature of the fluid and a decrease in the rate of heat transfer and skin friction. The researchers also observed that there was an increase in the velocity and temperature of the fluid with increase in the Grashof number but a decrease in Viscous variation parameter. From the

research work cited above, it is observed that extensive research work has been carried out on MHD natural convection fluid flow past a surface. However, no emphasis has been given to the problem studied by Molla et. al (2012) considering unsteady flow where viscosity is taken as a linear function of temperature . Therefore, this work presents findings of studies on the effects of variable viscosity on unsteady natural convection hydromagnetic flow past an isothermal sphere taking viscosity as linear function of temperature and analysis of the results using Direct Numerical Scheme.
In the next chapter, the governing equations, assumptions made in this study, Mathematical formulation and Methodology used in this study are discussed.

Chapter 3

METHODOLOGY

3.1 Introduction

The equations governing the flow of an incompressible electrically conducting fluid in the presence of transverse magnetic field lines past an isothermal sphere are presented in this chapter. First, the assumptions used in this study are highlighted. The equations of conservation of mass, equation of motion and equation of energy are considered in general forms, non-dimensional parameters are defined and non-dimensionalization of the resulting equations is also discussed. The Direct Numerical Scheme is discussed together with the finite difference scheme.

3.2 Assumptions

The following assumptions are made in this study;

i. The fluid is incompressible i.e. fluid density ρ is assumed to be a constant.

ii. The fluid flow is unsteady i.e. fluid velocity changes with time.

iii. Electrical conductivity and thermal conductivity are constant.

iv. All velocities are small compared with that of light $v^2/c^2 << 1$. Note that c in this case is the velocity of sound.

v. The external electric field and induced magnetic field due to polarization charges are negligible.

vi. The force due to electric field is negligible compared to the force $\hat{J} \times \hat{B}$ due to magnetic field.

3.3 The Governing Equations

3.3.1 Equation of conservation of mass

The law of conservation of mass states that mass can neither be created or destroyed and it forms the basis for the equation of continuity. This equation is derived by taking a mass balance of fluid entering and leaving a volume in the flow field. From Onyango and Surindar (2015), the equation of conservation of mass is given by;

$$\frac{\partial \rho}{\partial t} + \hat{\bigtriangledown} \cdot (\rho \hat{q}) = 0 \tag{3.1}$$

where $\hat{q}$ is the velocity in x, y and z direction.

3.3.2 Equation of motion

This equation describe the behaviour of physical system in terms of its motion as a function of time where the dynamics and kinematics of the motion are considered. In dynamics, momenta, forces and energy are put into consideration whereas kinematics deals with variables derived from the positions of objects and time. From Onyango and Surindar (2015), the expression of the equation is given as;

$$\frac{\partial \hat{q}}{\partial t} + \hat{q}(\hat{\bigtriangledown} \cdot \hat{q}) = -\frac{1}{\rho}\hat{\bigtriangledown} P + \vartheta \bigtriangledown^2 \hat{q} + \hat{F} \tag{3.2}$$

where $\frac{\partial \hat{q}}{\partial t}$ is the temporal acceleration, $\hat{q}(\hat{\bigtriangledown}\hat{q})$ is the convective term, $\hat{\bigtriangledown} P$ is the pressure gradient, $\vartheta \bigtriangledown^2 \hat{q}$ is the force due to viscosity and $\hat{F}$ represents the body forces vector in x ,y and z directions.

3.3.3 The Energy Equation

This equation is derived by applying the first law of thermodynamics to an arbitrary control volume in the flow field.From (Mwangi and Surindar, 2016), the equation is given as:

$$\rho C_p \left(\hat{u}\frac{\partial T}{\partial \hat{x}} + \hat{v}\frac{\partial T}{\partial \hat{y}} \right) = K \left(\frac{\partial^2 T}{\partial \hat{x}^2} + \frac{\partial^2 T}{\partial \hat{y}^2} \right) + \mu\phi \tag{3.3}$$

where the viscous dissipation term ϕ is defined as:

$$\phi = 2\left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 \tag{3.4}$$

3.4 Non-Dimensional Numbers

Dimensionless numbers which are also called the non-dimensional parameters are obtained by dividing the inertia force which is always present when a mass is in motion by viscous force or gravity force or pressure force or surface tension or elastic force. These parameters allow the application of the results obtained in a model to any other dynamically similar case. In this study, there are three numbers that are used. These numbers are Grashof Number,Gr, Prandtl Number, Pr, and Reynolds number, Re.

3.4.1 Grashof Number

This number occurs in natural convection problem and is usually defined as:

$$Gr = \frac{g\beta \left(T_w - T_\infty \right) a^3}{\vartheta^2} \tag{3.5}$$

Where g is acceleration due to gravity,β is the coefficient of Thermal Expansion,a is the radius of the sphere where in this study, it is the characteristic length of the problem,ϑ is the Reference Kinematic Viscosity,T_w is the temperature of the sphere and T_∞ is the temperature of the ambient fluid. This number gives the relative importance of buoyancy force to viscous force.

3.4.2 Prandtl Number

This number gives the ratio of viscous force to the thermal force and is defined as:

$$Pr = \frac{\mu C_p}{K} = \frac{\vartheta}{\left(\frac{K}{\rho C_p}\right)} \tag{3.6}$$

Where $\left(\frac{K}{\rho C_p}\right)$ is the thermal diffusivity and ϑ is the Reference kinematic Viscosity. Fluids that are more viscous have a higher value of ϑ and thus it follows that they have large Prandtl number. Prandtl number (Pr) is large when K is small and small when viscosity is small. This number gives the relative importance of viscous dissipation to thermal dissipation.

3.4.3 Reynolds number

The Reynolds number is defined as the ratio of inertial forces to viscous forces and consequently shows the relative importance of these two types of forces for given flow conditions. It signifies the relative predominance of the inertia to the viscous forces occuring in the flow systems. It is given as: $Re = \frac{\rho V L}{\mu}$.
This number is taken as the criterion of dynamic similarity in the flow situations where the viscous forces predominate; examples being; motion of submarine completely under water, low velocity motion around automobiles and aeroplanes, Incompressible flow through pipes of smaller sizes and flow through low speed turbomachines.

3.5 Mathematical Formulation

3.5.1 Equation of Conservation of Mass

Fluid flow is considered to be unsteady, incompressible and two dimensional laminar flow. Therefore, Density (ρ) is a constant and thus the term $\dfrac{\partial \rho}{\partial t} = 0$.
Thus, equation (3.1) reduces to $\hat{\bigtriangledown} \cdot (\rho \hat{q}) = 0$. Substituting the values of $\hat{\bigtriangledown}$ and $\hat{q}$, this equation becomes;

$$\rho \left(\hat{i}\frac{\partial}{\partial x^\star} + \hat{j}\frac{\partial}{\partial y^\star} + \hat{k}\frac{\partial}{\partial z^\star} \right) \cdot \left(\hat{i}u^\star + \hat{j}v^\star + \hat{k}w^\star \right) = 0 \tag{3.7}$$

which reduces to;

$$\rho \left(\frac{\partial u^\star}{\partial x^\star} + \frac{\partial v^\star}{\partial y^\star} + \frac{\partial w^\star}{\partial z^\star} \right) = 0 \tag{3.8}$$

But $r^\star(x) = aSin(x^\star)$ is the radial distance from the symmetric axis to the surface of the sphere. This term is included in equation (3.8) which becomes;

$$\rho \left(\frac{\partial r u^\star}{\partial x^\star} + \frac{\partial r v^\star}{\partial y^\star} + \frac{\partial r w^\star}{\partial z^\star} \right) = 0 \tag{3.9}$$

Since the flow is two dimensional, the term $\dfrac{\partial r w^\star}{\partial z^\star} = 0$. Thus, equation (3.9) becomes;

$$\rho \left(\frac{\partial r u^\star}{\partial x^\star} + \frac{\partial r v^\star}{\partial y^\star} \right) = 0 \tag{3.10}$$

Dividing (3.10) by ρ both sides gives;

$$\frac{\partial r u^\star}{\partial x^\star} + \frac{\partial r v^\star}{\partial y^\star} = 0 \tag{3.11}$$

Equation (3.11) is the dimensional equation of conservation of mass governing the flow.

3.5.2 Equation of Motion

From equation (3.2), the term $\dfrac{\partial \hat{q}}{\partial t} \neq 0$ since the fluid flow is unsteady.The term $\dfrac{\partial \hat{q}}{\partial t} = \dfrac{\partial (u^\star \hat{i} + v^\star \hat{j} + w^\star \hat{k})}{\partial t^\star}$. This will be given as $\dfrac{\partial \hat{q}}{\partial t} = \dfrac{\partial u^\star}{\partial t^\star}$ since the flow velocity varies with time and is considered to flow along the normal X-axis.

The term ;

$$\hat{q}(\hat{\bigtriangledown} \cdot \hat{q}) = u^{\star}\frac{\partial u^{\star}}{\partial x^{\star}} + v^{\star}\frac{\partial u^{\star}}{\partial y^{\star}} \tag{3.12}$$

since the flow is two dimensional.

For Laminar fluid flow; $\hat{\bigtriangledown} P = -\frac{\partial P}{\partial x}$ and $\frac{\partial \tau}{\partial y^{\star}} = \frac{\partial P}{\partial x}$. But $\tau = \mu \frac{\partial u^{\star}}{\partial y^{\star}}$. This is because the pressure gradient in the direction of flow is equal to the shear gradient in the direction normal to the direction of flow. Therefore, the pressure gradient term becomes;

$$\hat{\bigtriangledown} P = \frac{\partial \tau}{\partial y^{\star}} = \frac{\partial}{\partial y^{\star}}\left(\mu \frac{\partial u^{\star}}{\partial y^{\star}}\right) \tag{3.13}$$

The term;

$$\vartheta \hat{\bigtriangledown} \hat{q} = \frac{\mu}{\rho}\left(\frac{\partial^2 u^{\star}}{\partial x^{\star 2}} + \frac{\partial^2 v^{\star}}{\partial y^{* 2}}\right) = 0 \tag{3.14}$$

The sphere is immersed in the fluid and it is subjected to an upward force which tends to lift it up. This force is known as Buoyancy which is the tendency of an immersed body to be lifted up in the fluid. Therefore, from bousinessq approximation, this force in this study is given as;

$$g\beta(T - T_{\infty}) Sin\left(\frac{x^{*}}{a}\right) \tag{3.15}$$

The fluid in consideration is a ferrofluid and thus the Magnetic Force $\hat{F}$ is given as;

$$\hat{F} = \hat{J} \times \hat{B} \tag{3.16}$$

The generalized Ohms law neglecting the hall effects can be expressed as:

$$\hat{J} = \delta_0 \left[E + (\hat{q} \times \hat{B})\right]. \tag{3.17}$$

. E in equation (3.17) is the thermal electric effect and it is equivalent to 0 as there is no applied electric internally. Therefore,equation (3.17) can be written as:

$$\hat{J} = \delta_0 \left[\hat{q} \times \hat{B}\right]. \tag{3.18}$$

The velocity and Magnetic fields are given as: $\hat{q} = (u, v, 0)$ and $\hat{B} = (0, 0, B_0)$, Therefore;

$$\hat{q} \times \hat{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u & v & 0 \\ 0 & 0 & B_0 \end{vmatrix} = B_0 v\hat{i} - B_0 u\hat{j} \tag{3.19}$$

Therefore, $\hat{J} = \delta_0 B_0 v\hat{i} - \delta_0 B_0 u\hat{j}$

$$\hat{J} \times \hat{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \delta_0 v B_0 & -\delta_0 u B_0 & 0 \\ 0 & 0 & B_0 \end{vmatrix} = -\delta_0 u B_0^2 \hat{i} - \delta_0 v B_0^2 \hat{j} \tag{3.20}$$

substituting equations (3.12),(3.13),(3.14),(3.15),(3.20) in the general equation of motion (3.2), the dimensional equation of motion governing the flow in this study becomes;

$$\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{\rho} \frac{\partial}{\partial y^*} \left(\mu \frac{\partial u^*}{\partial y^*} \right) + g\beta(T - T_\infty) Sin \left(\frac{x^*}{a} \right) - \frac{\delta_0 B_0^2 u^*}{\rho} \tag{3.21}$$

3.5.3 Energy Equation

From the general equation (3.3),$\mu\phi$ is the Viscous Energy dissipation term and it is very small since the sphere in consideration is isothermal (It is considered at constant temperature). Hence, it is neglected in this study.
The fluid flow is unsteady. Therefore, the term $\frac{\partial T^*}{\partial t^*}$ is included in this equation.
The term $(\hat{\bigtriangledown}^2 T^*) = \frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}}$ is considered along the y-axis due to boundary layer approximations where its considered that $\frac{\partial^2 T^*}{\partial x^{*2}} << \frac{\partial^2 T^*}{\partial y^{*2}}$
Therefore, the dimensional energy equation in this study is given as;

$$\frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{K}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} \tag{3.22}$$

Equations (3.11),(3.21)and (3.22) are the equations governing the flow with boundary conditions defined as;

$$\begin{aligned} &\hat{u} = \hat{v} = 0,\ \theta = 1 \text{ at } \hat{y} = 0 \\ &\hat{u} \to 0,\ \theta \to 0 \text{ as } \hat{y} \to \infty \end{aligned} \tag{3.23}$$

In the next section, non-dimensionalization of the governing equation (3.11), (3.21) and (3.22) is carried out.

3.6 Non-Dimensionalization

The principal use of dimensional analysis is to deduce from a study of the dimensions of the variables in any physical system certain limitations on the form of any possible relationship between those variables. The method is of great generality and mathematical simplicity. This is a process that starts with selecting a suitable scale against which all dimensions in a given physical model are scaled. This process aims at ensuring that the results obtained are applicable to other geometrically similar configurations under similar set of flow conditions.

The independent variables are non-dimensionalized according to the following dimensionless quantities;

$x = \frac{x^*}{a}$

$y = \frac{Gr^{\frac{1}{4}} y^*}{a}$

$u = \frac{\rho a}{\mu} Gr^{-\frac{1}{2}} u^*$

$v = \frac{\rho a}{\mu} Gr^{-\frac{1}{2}} v^*$

$\theta = \frac{T^* - T_\infty}{T_w - T_\infty}$

$t^* = \frac{ut}{a}$

The dimensionless quantities above together with the boundary condition defined in the above section are used in non-dimensionalization of the governing equations (3.11), (3.21) and (3.22)

Substituting the values of x^*, y^*, u^*, t^* and v^* with $x^* = xa, y^* = \frac{a}{Gr^{\frac{1}{4}}} y, u^* = \frac{\mu}{\rho a} Gr^{\frac{1}{2}} u, v^* = \frac{\mu}{\rho a} Gr^{\frac{1}{2}} v, T^* = \theta((T_w - T_\infty) + T_\infty)$. In equation (3.11) gives ;

$$\frac{\partial (ru^*)}{\partial x^*} = \frac{\partial \left(r \frac{\mu}{\rho a} Gr^{\frac{1}{2}} u \right)}{\partial (xa)} = \frac{\mu Gr^{\frac{1}{2}}}{\rho a^2} \frac{\partial (ru)}{\partial x} \qquad (3.24)$$

$$\frac{\partial (rv^*)}{\partial y^*} = \frac{\partial \left(r\frac{\mu}{\rho a} Gr^{\frac{1}{2}} v \right)}{\partial (\frac{a}{Gr^{\frac{1}{4}}} y)} = \frac{\mu Gr^{\frac{3}{4}}}{\rho a^2} \frac{\partial (rv)}{\partial y} \tag{3.25}$$

The equation becomes;

$$\frac{\mu Gr^{\frac{1}{2}}}{\rho a^2} \frac{\partial (ru)}{\partial x} + \frac{\mu Gr^{\frac{3}{4}}}{\rho a^2} \frac{\partial (rv)}{\partial y} = 0 \tag{3.26}$$

Multiply by $\dfrac{\rho a^2}{\mu Gr^{\frac{1}{2}}}$ both sides of equation (3.26),this equation reduces to;

$$\frac{\partial (ru)}{\partial x} + Gr^{\frac{1}{4}} \frac{\partial (rv)}{\partial y} = 0 \tag{3.27}$$

Equation (3.27) is the non-dimensionalized equation of conservation of mass.

Putting the values of x^*, y^*, u^*, T^* and v^* in equation (3.21),gives;

$$\frac{\partial u^*}{\partial t^*} = \frac{\partial \left(\frac{\mu Gr^{\frac{1}{2}}}{\rho a} \right) u}{\partial \left(\frac{Ut}{a} \right)} = \frac{\mu a Gr^{\frac{1}{2}}}{\rho a U} \frac{\partial u}{\partial t} \tag{3.28}$$

But $Re = \dfrac{\rho a U}{\mu}$,therefore, (3.28) becomes;

$$\frac{\partial u^*}{\partial t^*} = \frac{a Gr^{\frac{1}{2}}}{Re} \frac{\partial u}{\partial t} \tag{3.29}$$

$$u^* \frac{\partial u^*}{\partial x^*} = \frac{\mu Gr^{\frac{1}{2}}}{\rho a} u \frac{\partial \left(\frac{\mu Gr^{\frac{1}{2}}}{\rho a} u \right)}{\partial (xa)} = \frac{\mu^2 Gr}{\rho^2 a^3} u \frac{\partial u}{\partial x} \tag{3.30}$$

$$v^* \frac{\partial u^*}{\partial y^*} = \frac{\mu Gr^{\frac{1}{2}}}{\rho a} v \frac{\partial \left(\frac{\mu Gr^{\frac{1}{2}}}{\rho a} u \right)}{\partial \left(\frac{a}{Gr^{\frac{1}{4}}} \right) y} = \frac{\mu^2 Gr^{\frac{5}{4}}}{\rho^2 a^3} v \frac{\partial u}{\partial y} \tag{3.31}$$

$$\frac{1}{\rho} \frac{\partial \left(\mu \frac{\partial u^*}{\partial y^*} \right)}{\partial y^*} = \frac{1}{\rho} \frac{\partial \left(\mu_\infty (1+\gamma\theta) \right) \frac{\partial u^*}{\partial y^*}}{\partial y^*} = \frac{\mu_\infty \gamma}{\rho} \frac{\partial \theta}{\partial y^*} \frac{\partial u^*}{\partial y^*} + \frac{\mu_\infty (1+\gamma\theta)}{\rho} \frac{\partial^2 u^*}{\partial y^*} \tag{3.32}$$

From (3.32),the terms reduces to;

$$\frac{\mu_\infty \gamma}{\rho}\frac{\partial \theta}{\partial y^*}\frac{\partial u^*}{\partial y^*} = \frac{\mu_\infty \gamma}{\rho}\frac{\partial \theta}{\partial\left(\frac{a}{Gr^{\frac{1}{4}}}y\right)}\frac{\partial\left(\frac{\mu Gr^{\frac{1}{2}}}{\rho a}u\right)}{\partial\left(\frac{a}{Gr^{\frac{1}{4}}}y\right)} = \frac{\mu_\infty \mu \gamma Gr}{\rho^2 a^3}\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} \tag{3.33}$$

$$\frac{\mu_\infty(1+\gamma\theta)}{\rho}\frac{\partial^2 u^*}{\partial y^*} = \frac{\mu_\infty(1+\gamma\theta)}{\rho}\frac{\partial^2\left(\frac{\mu Gr^{\frac{1}{2}}}{\rho a}u\right)}{\partial\left(\frac{a}{Gr^{\frac{1}{4}}}\right)y^2} = \frac{\mu_\infty(1+\gamma\theta)\mu Gr}{\rho^2 a^3}\frac{\partial^2 u}{\partial y^2} \tag{3.34}$$

$$g\beta(T-T_\infty)Sin\left(\frac{x^*}{a}\right) = g\beta(T-T_\infty)Sin\left(\frac{xa}{a}\right) = g\beta(T-T_\infty)Sinx \tag{3.35}$$

$$\frac{\delta_0 B_0^2}{\rho}u^* = \frac{\delta_0 B_0^2}{\rho}\left(\frac{\mu Gr^{\frac{1}{2}}}{\rho a}u\right) = \frac{\delta_0 B_0^2 \mu Gr^{\frac{1}{2}}}{\rho^2 a}u \tag{3.36}$$

Substituting equations (3.29),(3.30),(3.31),(3.33),(3.34),(3.35)and (3.36) in (3.21) and multiplying the equation by $\frac{\rho^2 a^3}{\mu^2 Gr}$, the equation becomes;

$$\frac{Rea^2}{Gr^{\frac{1}{2}}U^2}\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+Gr^{\frac{1}{4}}v\frac{\partial u}{\partial y} = (1+\gamma\theta)\frac{\partial^2 u}{\partial y^2}+\gamma\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y}+\frac{\rho^2 a^3 g\beta(T-T_\infty)}{\mu^2}Sin(x)-\frac{\delta_0 B_0^2 \mu Gr^{\frac{1}{2}}}{\rho^2 a}u \tag{3.37}$$

But $Gr = \frac{\rho^2 a^3 g\beta(T_w-T_\infty)}{\mu^2}$,$\theta = \frac{T^*-T_\infty}{T_w-T_\infty}$ and $M = \frac{\delta_0 B_0^2 a^2}{\mu Gr^{\frac{1}{2}}}$. Therefore, substituting the value of Gr,θ and M in (3.37), the equation becomes;

$$\frac{Rea^2}{Gr^{\frac{1}{2}}U^2}\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + Gr^{\frac{1}{4}}v\frac{\partial u}{\partial y} = (1+\gamma\theta)\frac{\partial^2 u}{\partial y^2} + \gamma\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} + \theta Sin(x) - Mu \tag{3.38}$$

Equation (3.38) is the non-dimensionalized equation of motion.

Substituting the values of x^*, y^*, u^*, t^*, T^* and v^* in equation (3.23) gives;

$$\frac{\partial T^*}{\partial t^*} = \frac{\partial(\theta(T_w-T_\infty)+T_\infty)}{\partial\left(\frac{Ut}{a}\right)} = \frac{a(T_w-T_\infty)}{U}\frac{\partial \theta}{\partial t} \tag{3.39}$$

$$u^*\frac{\partial T^*}{\partial x^*} = \frac{\mu Gr^{\frac{1}{2}}(T_w-T_\infty)}{\rho a^2}u\frac{\partial \theta}{\partial x} \tag{3.40}$$

$$v^*\frac{\partial T^*}{\partial y^*} = \frac{\mu Gr^{\frac{1}{2}}(T_w-T_\infty)}{\rho a^2}v\frac{\partial \theta}{\partial y} \tag{3.41}$$

$$\frac{K}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} = \frac{K}{\rho C_p}\frac{\partial^2(\theta(T_w - T_\infty) + T_\infty)}{\partial\left(\frac{a}{Gr^{\frac{1}{4}}}y\right)^2} = \frac{K(T_w - T_\infty)Gr^{\frac{1}{4}}}{\rho a^2 C_p}\frac{\partial^2\theta}{\partial y^2} \tag{3.42}$$

Substituting equations (3.39),(3.40),(3.41)and(3.42) in equation (3.22) and multiplying by $\dfrac{\rho a^2}{\mu Gr^{\frac{1}{2}}(T_w - T_\infty)}$, this equation becomes;

$$\frac{\rho a^3}{\mu Gr^{\frac{1}{2}}U}\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + Gr^{\frac{1}{4}}v\frac{\partial\theta}{\partial y} = \frac{K}{\mu C_p}\frac{\partial^2\theta}{\partial y^2} \tag{3.43}$$

But $Re = \dfrac{\rho a U}{\mu}$ and $Pr = \dfrac{\mu C_p}{K}$, therefore, equation (3.43) can be written as;

$$\frac{Rea^2}{Gr^{\frac{1}{2}}U^2}\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + Gr^{\frac{1}{4}}v\frac{\partial\theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2} \tag{3.44}$$

Equation (3.44) is the non-dimensionalized equation of energy. The initial and boundary conditions from the previous section obtained using the non-dimensional parameters becomes;

$$u = v = 0,\ \theta = 1,\ y = 0 \text{ at } t = 0 \tag{3.45}$$

$$\begin{aligned} &u = v = 0,\ \theta = 1,\ y = 0 \text{ at } \quad \text{any } t \\ &u \to 0,\ \theta \to 0 \text{ as } y \to \infty \end{aligned} \tag{3.46}$$

The process of non-dimensionalization helps in writing the important parameters in a problem as a functional relationship between them. The governing equations (3.27),(3.37) and (3.44) in their non-dimensional form together with the boundary conditions above are presented in finite differences which are consistent with the method of solution in the next section.

3.7 Direct Numerical Scheme(DNS)

To apply the Direct Numerical Scheme method, a new set of transformations are introduced. These are;

$X = x$, $Y = y$, $U = \dfrac{u}{x}$, $V = \dfrac{v}{y}$. Therefore, $u = Ux = UX$ and $v = Vy = VY$

But $r(\hat{x})$ is the radial distance from the centre of the sphere in consideration. The non-dimensionalized equation of continuity can be written as follows using the transformations above;

$$\frac{\partial(ru)}{\partial x} = \frac{\partial(UXSinX)}{\partial X} = XSinX\frac{\partial U}{\partial X} + USinX + UXCosX \tag{3.47}$$

$$Gr^{\frac{1}{4}}\frac{\partial(rv)}{\partial y} = Gr^{\frac{1}{4}}\frac{\partial(VYSinX)}{\partial Y} = Gr^{\frac{1}{4}}YSinX\frac{\partial V}{\partial Y} + Gr^{\frac{1}{4}}VSinX \tag{3.48}$$

Therefore, Replacing equations (3.47)and(3.48) in equation continuity, we obtain;

$$XSinX\frac{\partial U}{\partial X} + USinX + UXCosX + Gr^{\frac{1}{4}}VSinX + Gr^{\frac{1}{4}}YSinX\frac{\partial V}{\partial Y} = 0 \tag{3.49}$$

Dividing equation (3.49) by $SinX$, the equation can be written as;

$$X\frac{\partial U}{\partial X} + \left[1 + X\frac{CosX}{SinX}\right]U + Gr^{\frac{1}{4}}V + Gr^{\frac{1}{4}}Y\frac{\partial V}{\partial Y} = 0 \tag{3.50}$$

Substituting the transformations above in the non-dimensionalized equation of motion, we obtain the following transformations;

$$\frac{Rea^2}{Gr^{\frac{1}{2}}U^2}\frac{\partial u}{\partial t} = \frac{Rea^2}{Gr^{\frac{1}{2}}U^2}\frac{\partial(UX)}{\partial T} = \frac{Rea^2}{Gr^{\frac{1}{2}}U^2}X\frac{\partial U}{\partial T} \tag{3.51}$$

$$u\frac{\partial u}{\partial x} = UX\frac{\partial(UX)}{\partial X} = UX^2\frac{\partial U}{\partial X} + U^2X \tag{3.52}$$

$$Gr^{\frac{1}{4}}v\frac{\partial u}{\partial y} = Gr^{\frac{1}{4}}VY\frac{\partial(UX)}{\partial Y} = Gr^{\frac{1}{4}}VXY\frac{\partial U}{\partial Y} \tag{3.53}$$

$$\gamma\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y} = \gamma Gr^{\frac{1}{2}}\frac{\partial\theta}{\partial Y}\frac{\partial(UX)}{\partial Y} = \gamma X\frac{\partial\theta}{\partial Y}\frac{\partial U}{\partial Y} \tag{3.54}$$

$$(1+\gamma\theta)\frac{\partial^2 u}{\partial y^2} = (1+\gamma\theta)\frac{\partial^2(UX)}{\partial Y^2} = (1+\gamma\theta)X\frac{\partial^2 U}{\partial Y^2} \tag{3.55}$$

$$\theta Sinx = \theta SinX \tag{3.56}$$

$$Mu = MUX \tag{3.57}$$

Substituting equations(3.51),(3.52),(3.53),(3.54)(3.55),(3.56) and (3.57) in the equation of motion and dividing both sides of the equation by X,we obtain;

$$\frac{Rea^2}{Gr^{\frac{1}{2}}U^2}\frac{\partial U}{\partial T} + UX\frac{\partial U}{\partial X} + U^2 + Gr^{\frac{1}{4}}VY\frac{\partial U}{\partial Y} = (1+\gamma\theta)\frac{\partial^2 U}{\partial Y^2} + \gamma\frac{\partial\theta}{\partial Y}\frac{\partial U}{\partial Y} + \theta\frac{SinX}{X} - MU \tag{3.58}$$

Putting the transformations $X = x$, $Y = y$, $U = \frac{u}{x}$, $V = \frac{v}{y}$ and $t = T(Time)$ in the energy equation, the following equations are obtained;

$$\frac{Rea^2}{Gr^{\frac{1}{2}}U^2}\frac{\partial\theta}{\partial t} = \frac{Rea^2}{Gr^{\frac{1}{2}}U^2}\frac{\partial\theta}{\partial T} \tag{3.59}$$

$$u\frac{\partial\theta}{\partial x} = UX\frac{\partial\theta}{\partial X} \tag{3.60}$$

$$Gr^{\frac{1}{4}}v\frac{\partial\theta}{\partial y} = Gr^{\frac{1}{4}}VY\frac{\partial\theta}{\partial Y} \tag{3.61}$$

$$\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial Y^2} \tag{3.62}$$

Replacing equations (3.58),(3.59),(3.60) and (3.61) in the non- dimensionalized equation of energy,the equation becomes;

$$\frac{Rea^2}{Gr^{\frac{1}{2}}U^2}\frac{\partial\theta}{\partial T} + UX\frac{\partial\theta}{\partial X} + Gr^{\frac{1}{4}}VY\frac{\partial\theta}{\partial Y} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial Y^2} \tag{3.63}$$

From Molla et al. (2012),in order to determine the physical quantities, namely the shearing stress and the rate of heat transfer the following dimensionless relations are used in this study:

$$\frac{C_f Gr^{\frac{1}{4}}}{2(1+\gamma)} = X\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \tag{3.64}$$

$$NuGr^{\frac{-1}{4}} = -\left(\frac{\partial\theta}{\partial Y}\right)_{Y=0} \tag{3.65}$$

Using the transformations $X = x$, $Y = y$, $U = \frac{u}{x}$, $V = \frac{v}{y}$,the initial and boundary conditions represented in the previous section can be written as;

Initial Conditions;

$$U = V = 0,\ \theta = 1 \text{ at } T = 0 \text{ any } X \text{ and } Y \tag{3.66}$$

Boundary Conditions

$$\begin{aligned} &U = V = 0,\ \theta = 1 \text{ at } X = 0 \text{ any } Y, \text{ for all } T \\ &U = V = 0,\ \theta = 1 \text{ at } Y = 0,\ X > 0, \text{ for all } T \\ &U \to 0,\ \theta \to 0 \text{ as } Y \to \infty,\ X > 0, \text{ for all } T \end{aligned} \tag{3.67}$$

Equations (3.50),(3.58),(3.63),(3.64)and (3.65) together with the boundary conditions above are written in finite differences as explained in the next section.

3.8 Finite Difference Technique

The finite difference is a technique used for solving differential equations by using the difference equations in order to approximate. In mathematics, finite-difference methods (FDM) are numerical methods for solving differential equations by approximating them with difference equations, in which finite differences approximate the derivatives. FDMs are thus discretization methods.

The finite difference approximations for derivatives is one of the methods that can be used to solve differential equations. The finite difference method is close to numerical schemes which are used to solve ordinary and partial differential equations. This consists of approximating the differential operator when the derivative in the equations are replaced by difference quotients. The domain is subdivided in space and time and approximations of the solution are computed at the space or time points.

In this method, the (t, y) plane is divided into a network of rectangles of sides $\triangle t = h$ and $\triangle y = k$ by drawing the set of lines $t = ih$ and $y = jk$ where $i, j = 0, 1, 2$. Each nodal point is identified by double index (i, j) that define its location with respect to t and y as shown in the figure below:

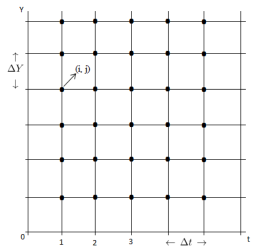

Figure 3.1: Finite Difference Mesh

From (3.1),consider a reference point (i, j) where i and j represent t and x respectively. Using the notation (i) for (t) and (j) for (x) we define the adjacent points that are i and j units from the reference point and give their co-ordinates in terms of(i) and $(\triangle t)$ along the x-axis, j and $(\triangle x)$ along the y-axis. In finite difference approximation the derivatives are replaced with the finite differences.$U = u(t, x)$ and the points of intersection of these families of lines are called mesh points or grid points. Then, by central differencing, the first and second order derivatives with respect to t are obtained in finite difference form. A finite difference mesh is used to express the unknown functional values at the interior mesh using the known boundary points. Crank Nicolson proposed a method in 1957 in which the second derivative is replaced by the average of the finite difference approximation on the and the row thus we have the proposed averages as

$$\frac{\partial U}{\partial T} = \frac{U_{i,j}^{k+1} - U_{i,j}^{k}}{2(\triangle T)} \tag{3.68}$$

$$\frac{\partial U}{\partial X} = \frac{U_{i+1,j}^{k} - U_{i-1,j}^{k}}{2(\triangle X)} \tag{3.69}$$

$$\frac{\partial U}{\partial Y} = \frac{U_{i,j+1}^{k} - U_{i,j-1}^{k}}{2(\triangle Y)} \tag{3.70}$$

$$\frac{\partial V}{\partial Y} = \frac{V_{i,j+1}^{k} - V_{i,j-1}^{k}}{2(\triangle Y)} \tag{3.71}$$

$$\frac{\partial^2 U}{\partial Y^2} = \frac{U_{i,j+1}^{k} - 2U_{i,j}^{k} + U_{i,j-1}^{k}}{(\triangle Y)^2} \tag{3.72}$$

$$\frac{\partial \theta}{\partial X} = \frac{\theta_{i+1,j}^{k} - \theta_{i-1,j}^{k}}{2(\triangle X)} \tag{3.73}$$

$$\frac{\partial \theta}{\partial Y} = \frac{\theta_{i,j+1}^{k} - \theta_{i,j-1}^{k}}{2(\triangle Y)} \tag{3.74}$$

$$\frac{\partial^2 \theta}{\partial Y^2} = \frac{\theta_{i,j+1}^{k} - 2\theta_{i,j}^{k} + \theta_{i,j-1}^{k}}{(\triangle Y)^2} \tag{3.75}$$

3.9 Governing Equations in Finite Difference Form

Equations (3.69),(3.70),(3.71),(3.72),(3.73),(3.74)and (3.75) are substituted for the derivatives in equations (3.50),(3.58),(3.63),(3.64)and (3.65). The derivatives are written in

Central Finite Difference Form as shown above.

Equation (3.49)becomes;

$$\left[\frac{U_{i+1,j}^{k}-U_{i-1,j}^{k}}{2(\triangle X)}\right]+U_{i,j}^{k}\left[1+X_i\frac{CosX_i}{SinX_i}\right]+Gr^{\frac{1}{4}}\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]+Gr^{\frac{1}{4}}Y_j\left[\frac{V_{i,j+1}^{k}-V_{i,j-1}^{k}}{2(\triangle Y)}\right]=0 \tag{3.76}$$

Making $U_{i,j}^{k}$ the subject of the formula in equation (3.76), we obtain;

$$U_{i,j}^{k}=-\left[\left[\frac{U_{i+1,j}^{k}-U_{i-1,j}^{k}}{2(\triangle X)}\right]+Gr^{\frac{1}{4}}\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]+Gr^{\frac{1}{4}}Y_j\left[\frac{V_{i,j+1}^{k}-V_{i,j-1}^{k}}{2(\triangle Y)}\right]\right]\div\left[1+X_i\frac{CosX_i}{SinX_i}\right] \tag{3.77}$$

Equation (3.58) can be written as;

$$\begin{aligned}\frac{Rea^2}{Gr^{\frac{1}{2}}U^2}\left[\frac{U_{i,j}^{k+1}-U_{i,j}^{k}}{2(\triangle T)}\right]+X_i\left[\frac{U_{i,j+1}^{k}+U_{i,j-1}^{k}}{2}\right]\left[\frac{U_{i+1,j}^{k}-U_{i-1,j}^{k}}{2(\triangle X)}\right]+\\ \left[\frac{U_{i,j+1}^{k}+U_{i,j-1}^{k}}{2}\right]^2+Gr^{\frac{1}{4}}Y_j\left[\frac{V_{i,j+1}^{k}-V_{i,j-1}^{k}}{2}\right]\left[\frac{U_{i,j+1}^{k}-U_{i,j-1}^{k}}{2(\triangle Y)}\right]+\left[\frac{U_{i+1,j}^{k}-U_{i-1,j}^{k}}{2}\right]^2=\\ (1+\gamma\theta_{i,j}^{k})\left[\frac{U_{i,j+1}^{k}-2U_{i,j}^{k}+U_{i,j-1}^{k}}{(\triangle Y)^2}\right]+\gamma\left[\frac{U_{i,j+1}^{k}-U_{i,j-1}^{k}}{2\triangle Y}\right]\left[\frac{\theta_{i,j+1}^{k}-\theta_{i,j-1}^{k}}{2\triangle Y}\right]+\theta_{i,j}^{k}\frac{SinX_i}{X_i}-MU_{i,j}^{k}\end{aligned} \tag{3.78}$$

From equation (3.78), making $U_{i,j}^{k+1}$ the subject of the formulae, this equation becomes;

$$\begin{aligned}U_{i,j}^{k+1}=\frac{Rea^2}{Gr^{\frac{1}{2}}U^2}\left[\frac{1}{2(\triangle T)}\right]U_{i,j}^{k}+(1+\gamma\theta_{i,j}^{k})\left[\frac{U_{i,j+1}^{k}-2U_{i,j}^{k}+U_{i,j-1}^{k}}{(\triangle Y)^2}\right]+\\ \gamma\left[\frac{U_{i,j+1}^{k}-U_{i,j-1}^{k}}{2\triangle Y}\right]\left[\frac{\theta_{i,j+1}^{k}-\theta_{i,j-1}^{k}}{2\triangle Y}\right]+\\ \theta_{i,j}^{k}\frac{SinX_i}{X_i}-MU_{i,j}^{k}-\left[X_i\left[\frac{U_{i,j+1}^{k}+U_{i,j-1}^{k}}{2}\right]\left[\frac{U_{i+1,j}^{k}-U_{i-1,j}^{k}}{2(\triangle X)}\right]\right]-\\ \left[\left[\frac{U_{i,j+1}^{k}+U_{i,j-1}^{k}}{2}\right]^2+Gr^{\frac{1}{4}}Y_j\left[\frac{V_{i,j+1}^{k}-V_{i,j-1}^{k}}{2}\right]\left[\frac{U_{i,j+1}^{k}-U_{i,j-1}^{k}}{2(\triangle Y)}\right]+\left[\frac{U_{i+1,j}^{k}-U_{i-1,j}^{k}}{2}\right]^2\right]\\ \div\frac{Rea^2}{Gr^{\frac{1}{2}}U^2}\left[\frac{1}{2(\triangle T)}\right]\end{aligned} \tag{3.79}$$

Writing equation (3.63) in Finite differences, we obtain;

$$\frac{Rea^2}{Gr^{\frac{1}{2}}U^2}\left[\frac{\theta_{i,j}^{k+1}-\theta_{i,j}^{k}}{2(\triangle T)}\right]+X_i\left[\frac{U_{i,j+1}^{k}+U_{i,j-1}^{k}}{2}\right]\left[\frac{\theta_{i+1,j}^{k}-\theta_{i-1,j}^{k}}{2(\triangle X)}\right]+ Gr^{\frac{1}{4}}Y_j\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]\left[\frac{\theta_{i,j+1}^{k}-\theta_{i,j-1}^{k}}{2(\triangle Y)}\right]= \frac{1}{Pr}\left[\frac{\theta_{i,j+1}^{k}-2\theta_{i,j}^{k}+\theta_{i,j-1}^{k}}{(\triangle Y)^2}\right] \quad (3.80)$$

From equation (3.79), making $\theta_{i,j}^{k+1}$ the subject of the formulae, gives;

$$\theta_{i,j}^{k+1}=\frac{Rea^2}{Gr^{\frac{1}{2}}U^2}\left[\frac{1}{2(\triangle T)}\right]\theta_{i,j}^{k}+\frac{1}{Pr}\left[\frac{\theta_{i,j+1}^{k}-2\theta_{i,j}^{k}+\theta_{i,j-1}^{k}}{(\triangle Y)^2}\right]-\left[X_i\left[\frac{U_{i,j+1}^{k}+U_{i,j-1}^{k}}{2}\right]\left[\frac{\theta_{i+1,j}^{k}-\theta_{i-1,j}^{k}}{2(\triangle X)}\right]\right]- \left[Gr^{\frac{1}{4}}Y_j\left[\frac{V_{i,j+1}^{k}+V_{i,j-1}^{k}}{2}\right]\left[\frac{\theta_{i,j+1}^{k}-\theta_{i,j-1}^{k}}{2(\triangle Y)}\right]\right]\div\frac{Rea^2}{Gr^{\frac{1}{2}}U^2}\left[\frac{1}{2(\triangle T)}\right] \quad (3.81)$$

The physical quantities to be obtained are the shearing stress(rate of skin friction) and the rate of heat transfer. The finite difference equations used to obtain these results are;

$$\frac{C_f Gr^{\frac{1}{4}}}{2(1+\gamma)}=X_i\left(\frac{U_{i,j+1}^{k}-U_{i,j-1}^{k}}{2(\triangle Y)}\right)_{Y=0} \quad (3.82)$$

$$NuGr^{\frac{-1}{4}}=-\left(\frac{\theta_{i,j+1}^{k}-\theta_{i,j-1}^{k}}{2(\triangle Y)}\right)_{Y=0} \quad (3.83)$$

Therefore, equations(3.77),(3.78),(3.80),(3.81) and (3.82) are the Final Set of Finite Difference Equations governing the fluid flow in this study and are solved using a computer code in MATLAB software.

The results obtained from the simulation of equations (3.77),(3.79),(3.81),(3.82) and (3.83) in the Computer code and discussions are presented in the next chapter.

Chapter 4

RESULTS AND DISCUSSIONS

4.1 Introduction

In this chapter, the results of the simulations are presented, followed by discussions at each step and validation of the results obtained.
In this study, an investigation of the effects of variable viscosity on unsteady natural convection hydromagnetic flow past an isothermal sphere has been carried out. Viscosity is taken as a linear function of Temperature in this study. The numerical solutions start from the lower stagnation point $x = 0$, round the sphere to the upper stagnation point where $x = \pi$. The Reynolds number is taken as; $Re(= 3, 4, 5)$, Grashof number, $Gr(= 50, 65, 85, 100)$, Magnetic parameter, $M(= 0, 0.25, 0.5, 1)$ and Viscous variation parameter, $\gamma(= 0, 0.25, 0.5, 1)$.

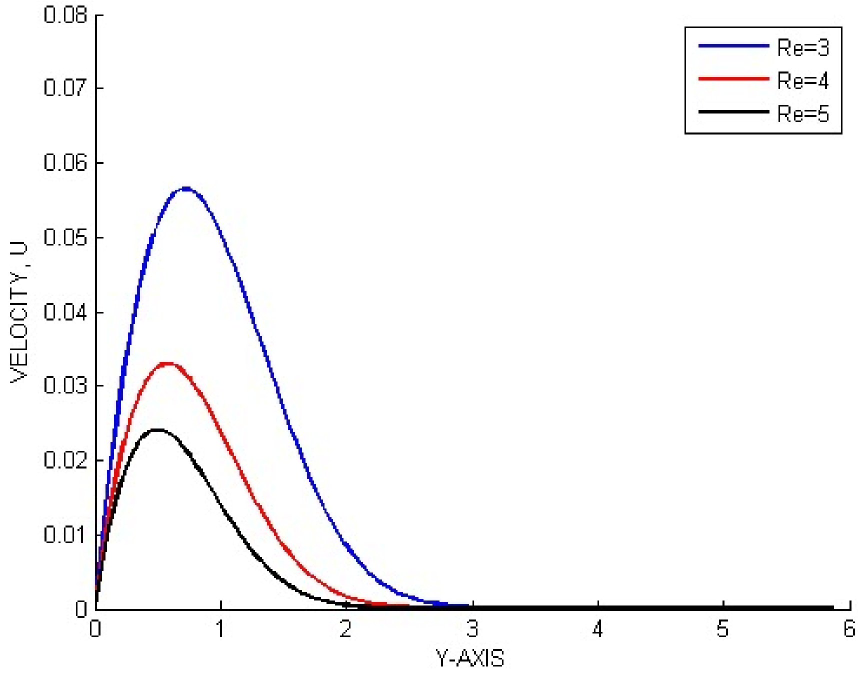

Figure 4.1: Primary Velocity (U)against Y-axis varying Reynolds number Re

Figure 4.1 above represents the primary velocity(U) against the Y-axis varying Reynolds number. The Magnetic parameter is taken as $M = 0.5$, $Gr = 85$,$\gamma = 0.5$ and $Re(= 3, 4, 5)$. It is observed that velocity profiles in this figure decreases with increase in Reynolds number and increases with decrease in Reynolds number. In this study, Re is considered as an inverse function and thus increase in Re leads to increase in the viscosity of the fluid and decrease in Re leads to decrease in the viscosity. Increasing viscosity leads to an increase in the viscous force that opposes the motion of the fluid and decrease in viscosity leads to a decrease in the viscous force. Therefore, in this case, it can be concluded that increase in Reynolds number leads to an increase in the viscosity of the fluid and thus the decrease in the velocity of the fluid which leads to decrease in the velocity profiles.

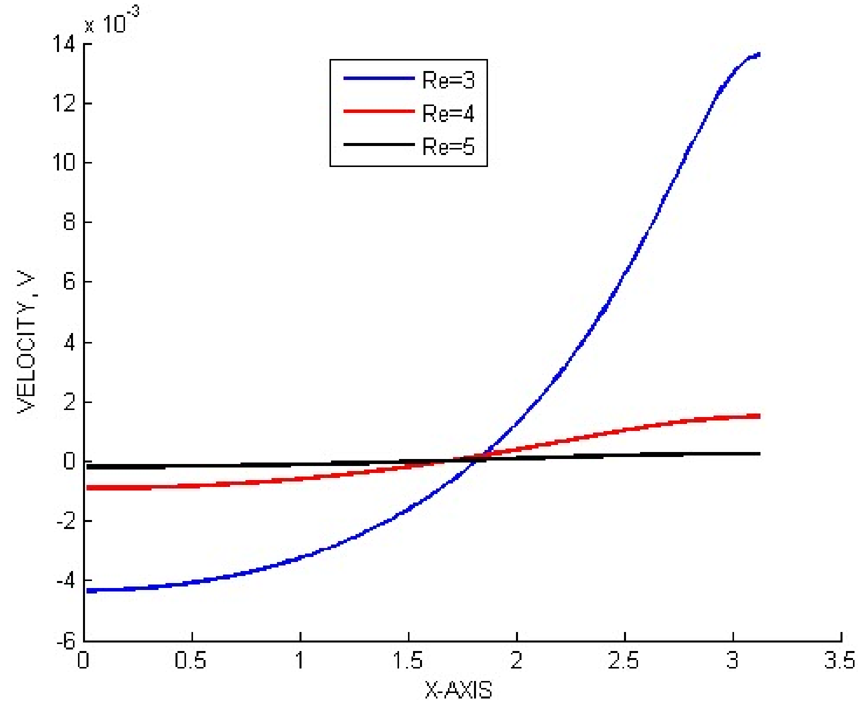

Figure 4.2: Secondary Velocity (V) against X-axis varying Reynolds number Re

Figure 4.2 above represents secondary velocity against the x-axis varying Reynolds number where $M = 0.5$, $Gr = 85$, $\gamma = 0.5$ and $Re(= 3, 4, 5)$. It is observed that the velocity starts from a negative value and increase to a positive value. Increase in Reynolds number leads to an increase in the velocity profiles until there is no change in the velocity profile at point (0). There is an intersection of the velocity profiles as observed from the above figure. This is because there is a circular motion displayed by the fluid which leads to a vortex flow. This leads to a change in the pressure gradient of the flow and thus the secondary flow of the fluid at the floor of the sphere changes as shown in the diagram above with increase in the Reynolds number.

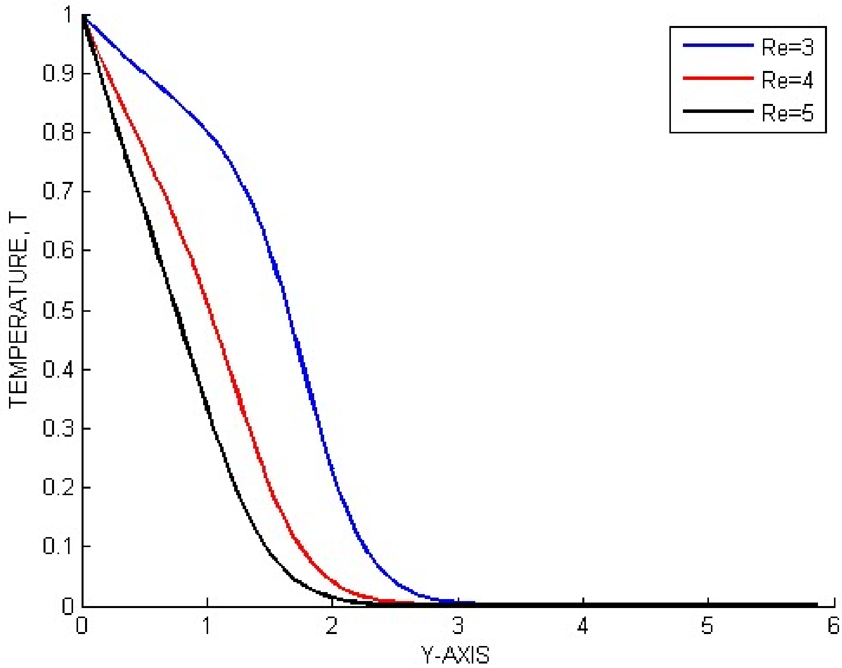

Figure 4.3: Temperature(T)against Y-axis varying Reynolds number Re

From figure 4.3 above, $M = 0.5$, $Gr = 85$, $\gamma = 0.5$ and $Re(= 3, 4, 5)$ and it is observed that increase in Reynolds number leads to decrease in the temperature profiles and decrease in Re leads to increase in the temperature profiles. In this study, Re is considered as an inverse function and thus increase in Re leads to increase in the viscosity of the fluid and decrease in Re leads to decrease in the viscosity. Increasing viscosity leads to an increase in the viscous force that opposes the motion of the fluid and decrease in viscosity leads to a decrease in the viscous force. Therefore, in this case, it can be concluded that increase in Reynolds number leads to an increase in the viscosity of the fluid and thus the decrease in the temperature of the fluid which leads to decrease in the temperature profiles.

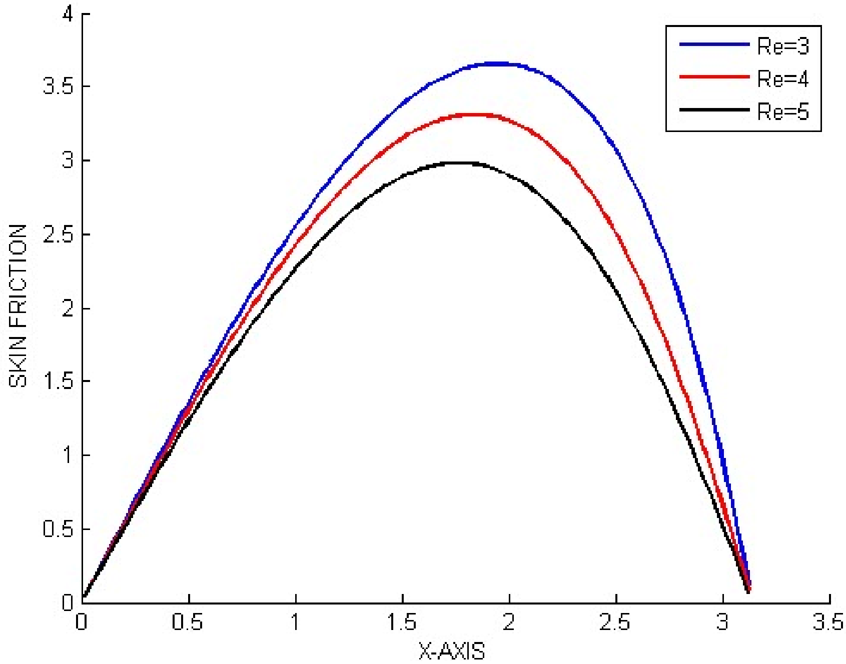

Figure 4.4: Skin Friction against X-axis varying Reynolds number Re

Figure 4.4 represents the skin friction against the X-axis varying Reynolds number while $M = 0.5$, $Gr = 85$ and $\gamma = 0.5$. It is observed that the skin-friction profiles decreases with increase in Reynolds number and increases with decrease in Reynolds number. Increase in Re leads to increase in the viscosity of the fluid which leads to increase in the viscous force due to the inverse nature of Reynolds number in this study. Increased viscous force leads to a reduction in the velocity of the fluid and thus the reduction in the velocity gradient. Decrease in velocity gradient leads to a decrease in the skin friction in the fluid as portrayed in the profiles above.

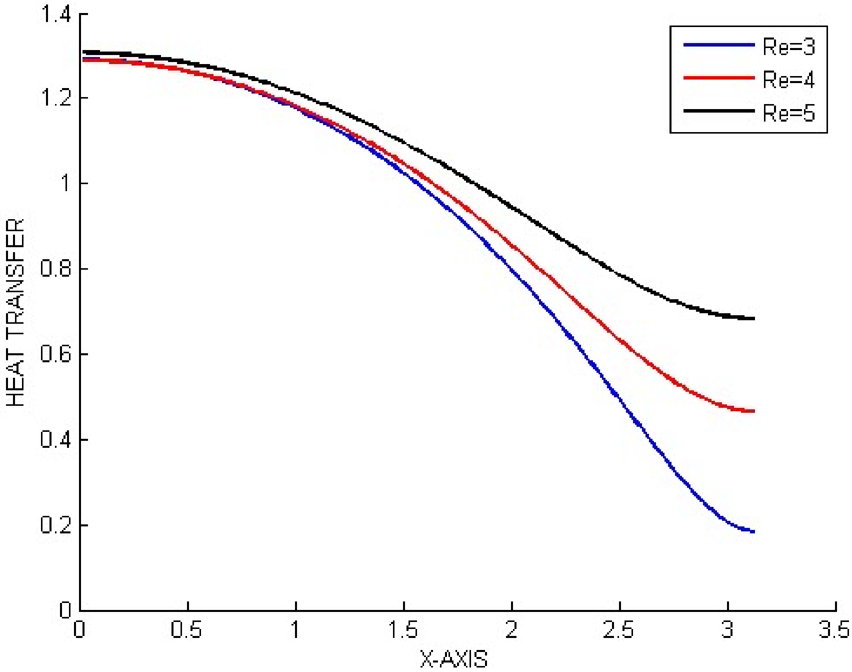

Figure 4.5: Heat Transfer against X-axis varying Reynolds number Re

Figure 4.5 above shows the Heat transfer against x-axis varying Reynolds number whereas $M = 0.5$, $Gr = 85$ and $\gamma = 0.5$. It is observed that increase in Reynolds number leads to an increase in the Heat transfer and decrease in Reynolds number leads to a decrease in the heat transfer. Increase in Re leads to increase in the viscosity of the fluid which leads to increase in the viscous force due to the inverse nature of Reynolds number in this study. Increased viscous force leads to a reduction in the temperature of the fluid and thus the reduction in the temperature gradient.

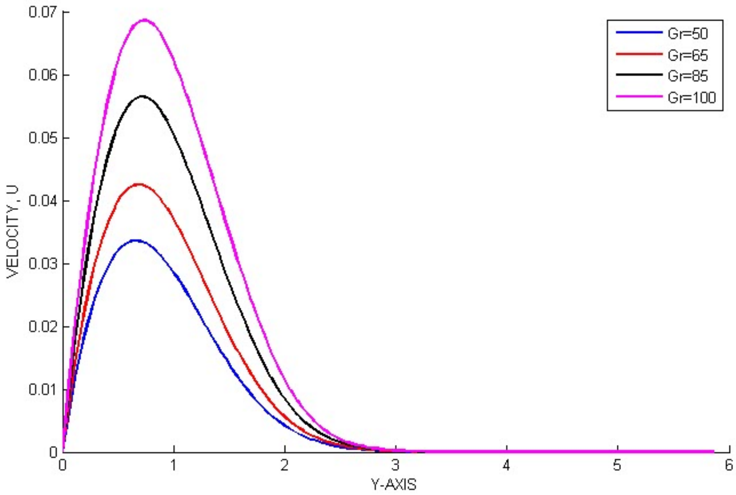

Figure 4.6: Primary Velocity against Y-axis for different values of Grashof number Gr

Figure 4.6 above represents the primary Velocity against Y-axis varying the Grashof number while $M = 0.5$, $Re = 4$ and $\gamma = 0.5$. It is observed that increase in Grashof number leads to an increase in the velocity profiles and decrease in Grashof number leads to decrease in the velocity profiles. Increase in Grashof number leads to decrease in the viscosity of the fluid and decrease in Grashof number leads to an increase in the viscosity of the fluid. Increase in the viscosity of the fluid leads to an increase in the viscous force which leads to a decrease in the velocity of the fluid and decrease in the viscosity leads to decrease in the viscous force thus increase in the velocity of the fluid. Therefore, increase in Grashof number leads to a decrease in Viscosity which reduces the viscous force and thus an increase in the velocity of the fluid as shown in the figure 4.6 above.

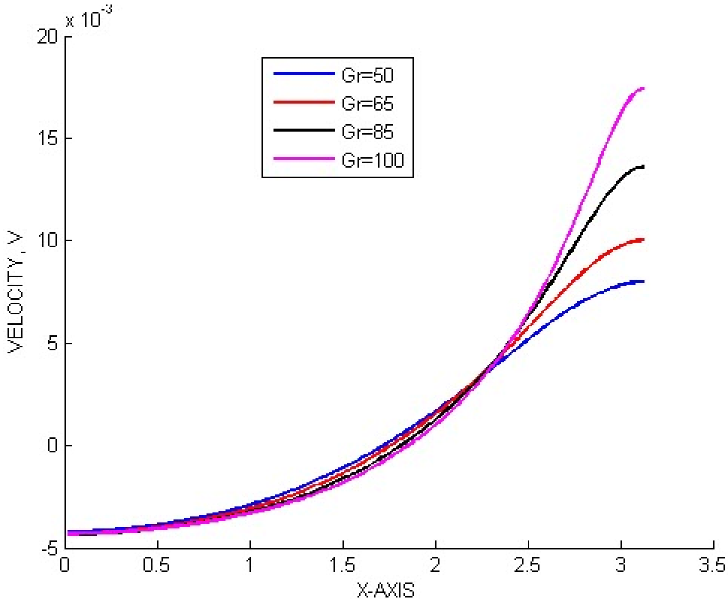

Figure 4.7: Secondary Velocity against Y-axis for different values of Grashof number (Gr)

From Figure 4.7 above, it is observed that there is a slight decrease in the velocity profiles when the Grashof number is increased. The profiles intersect at a point and begin to increase after the same point in an inverse behavior. This is because of the circular motion displayed by the fluid which leads to a vortex flow which causes a change in pressure gradient of the flow and thus the secondary flow of the fluid across the floor of the sphere.

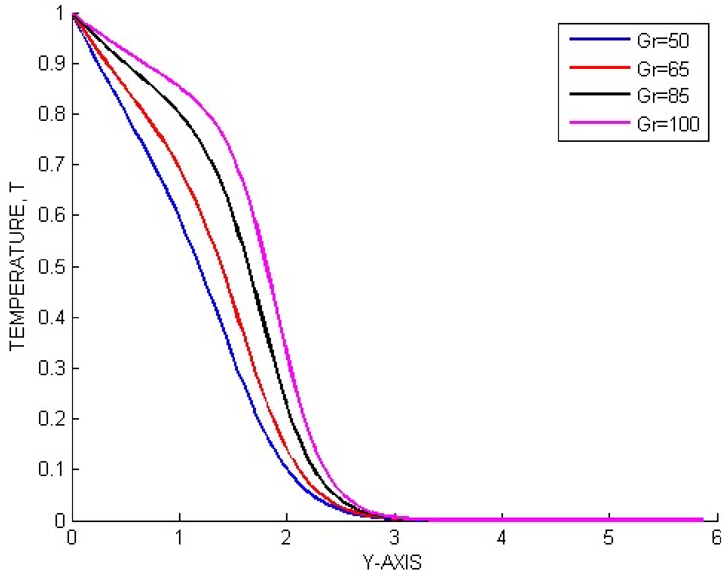

Figure 4.8: Temperature (T) against Y-axis for different values of Grashof number (Gr)

Figure 4.8 above shows the temperature (T) against the Y-axis varying Grashof number while $M = 0.5$, $Re = 4$ and $\gamma = 0.5$. It is observed that increase in Grashof number leads to an increase in the temperature profiles and decrease in Grashof number leads to a decrease in temperature profiles. Increase in Grashof number leads to decrease in the viscosity of the fluid and decrease in Grashof number leads to an increase in the viscosity of the fluid. Increase in the viscosity of the fluid leads to an increase in the viscous force which leads to a decrease in the temperature of the fluid and decrease in the viscosity leads to decrease in the viscous force thus increase in the temperature of the fluid. Therefore, increase in Grashof number leads to a decrease in Viscosity which reduces the viscous force and thus an increase in the temperature of the fluid and thus the increase in the temperature profiles.

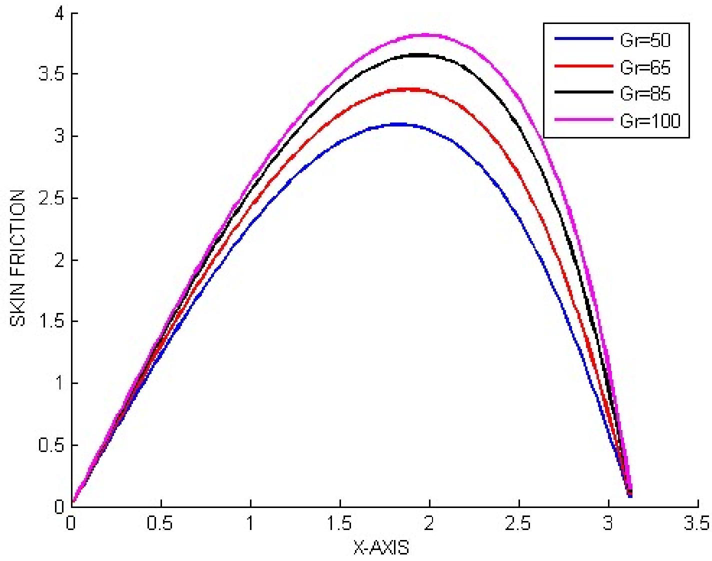

Figure 4.9: Skin Friction against X-axis for different values of Grashof number (Gr)

Figure 4.9 above shows the profiles of skin friction against X-axis varying Grashof number while $M = 0.5$, $Re = 4$ and $\gamma = 0.5$. It is observed that increase in Grashof number leads to an increase in the skin friction profiles and decrease in Grashof number leads to a decrease in skin friction. Increase in Grashof number leads to decrease in the viscosity of the fluid and decrease in Grashof number leads to an increase in the viscosity of the fluid. Increase in the viscosity of the fluid leads to an increase in the viscous force which leads to a decrease in the velocity of the fluid and decrease in the viscosity leads to decrease in the viscous force thus increase in the velocity of the fluid. Decrease in velocity leads to a decrease in the velocity gradient of the fluid and increase in the velocity leads to an increase in the velocity gradient of the fluid. Therefore, in this case, increase in Grashof number leads to an increase in the velocity gradient of the fluid which leads to an increase in the skin friction of the fluid.

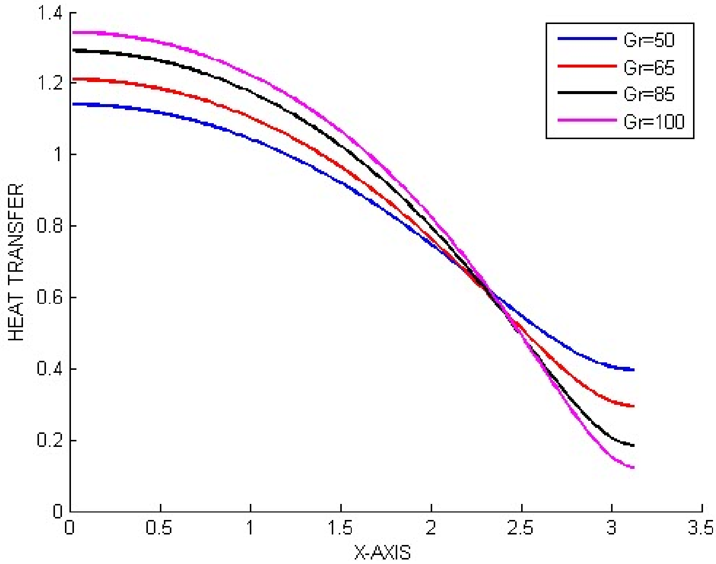

Figure 4.10: Heat Transfer against X-axis for different values of Grashof number (Gr)

Figure 4.10 above shows the profiles of Heat transfer against x-axis varying Grashof number while $M = 0.5$, $Re = 4$ and $\gamma = 0.5$. It is observed that increase in Grashof number leads to an increase in Heat transfer profiles and decrease in Grashof number leads to a decrease in the heat transfer. Increase in Grashof number leads to decrease in the viscosity of the fluid and decrease in Grashof number leads to an increase in the viscosity of the fluid. Increase in the viscosity of the fluid leads to an increase in the viscous force which leads to a decrease in the temperature of the fluid and decrease in the viscosity leads to decrease in the viscous force thus increase in the temperature of the fluid. Increase in temperature leads to an increase in temperature gradient of the fluid which leads to an increase in the heat transfer as shown in the figure above.

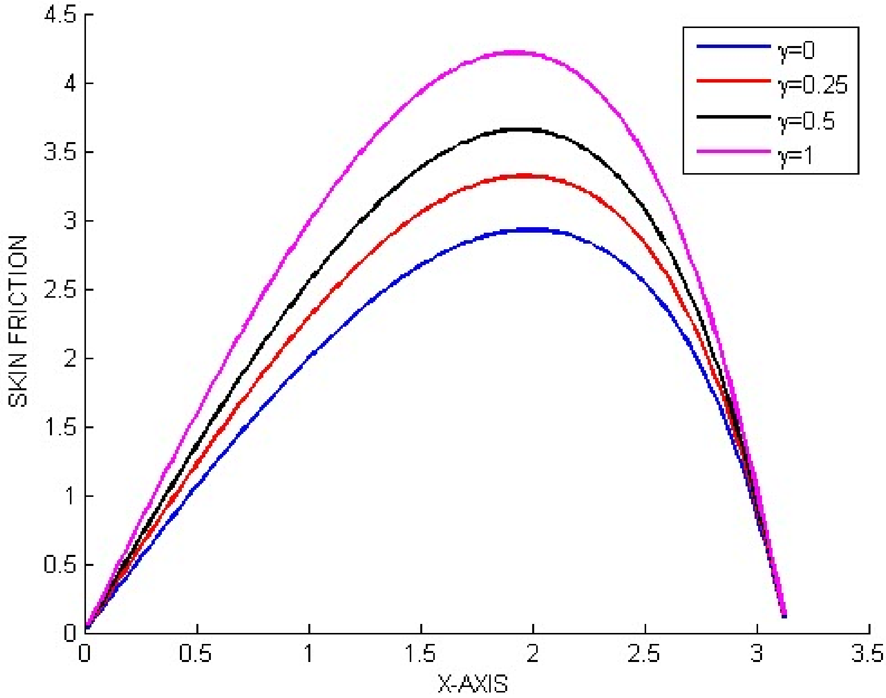

Figure 4.11: Primary Velocity against Y-axis for different values of Magnetic Parameter (M)

Figure 4.11 shows Primary velocity against Y-axis varying Magnetic parameter where $M(= 0, 0.25, 0.5.1.0)$ whereas $Gr = 85$, $Re = 4$ and $\gamma = 0.5$. It is observed that increase in Magnetic parameter leads to decrease in the velocity profiles and decrease in Magnetic parameter leads to an increase in the velocity profiles. Increase in Magnetic parameter leads to an increase in Lorentz force in the fluid which opposes the fluid flow and thus leading to a decrease in the velocity of the fluid thus the reduction in the velocity profiles shown above.

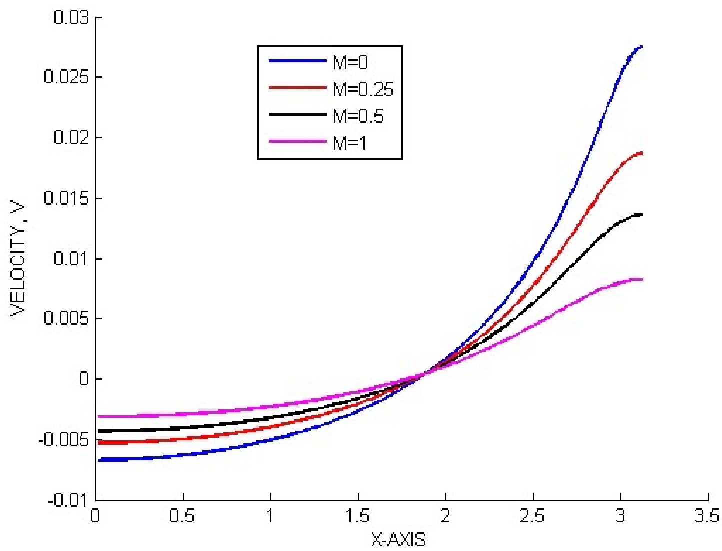

Figure 4.12: Secondary Velocity against X-axis for different values of Magnetic Parameter (M)

Figure 4.12 above shows the Secondary velocity (V) against x-axis varying magnetic parameter (M) whereas $Gr = 85$, $Re = 4$ and $\gamma = 0.5$. Increase in Magnetic parameter leads to an increase in the velocity profiles and decrease in the magnetic parameter leads to a decrease in the velocity profiles. There is an intersection of the velocity profiles at a certain point and then the profiles starts to rise. This is because as the fluid passes below the sphere, there is a circular motion that is displayed by the fluid which leads to a vortex flow. This makes the surface of the fluid to have a characteristic depression toward the axis of the spinning fluid.

At any elevation with the fluid, the pressure is greater near the surface of the sphere where the fluid is deeper than near the center of the sphere. The fluid pressure is greater where the speed of the fluid is slower and the pressure is a little less where the speed is faster and therefore, this is consistent with the Bernoullis principle. There is a pressure gradient from the perimeter of the sphere towards the center which leads to a centripetal force which is necessary for the circular motion of each parcel of the fluid. This pressure gradient accounts for the secondary flow of the boundary layer of the fluid flowing across the floor of the fluid. Thus, the variation of the velocity profiles with the magnetic parameter as shown above.

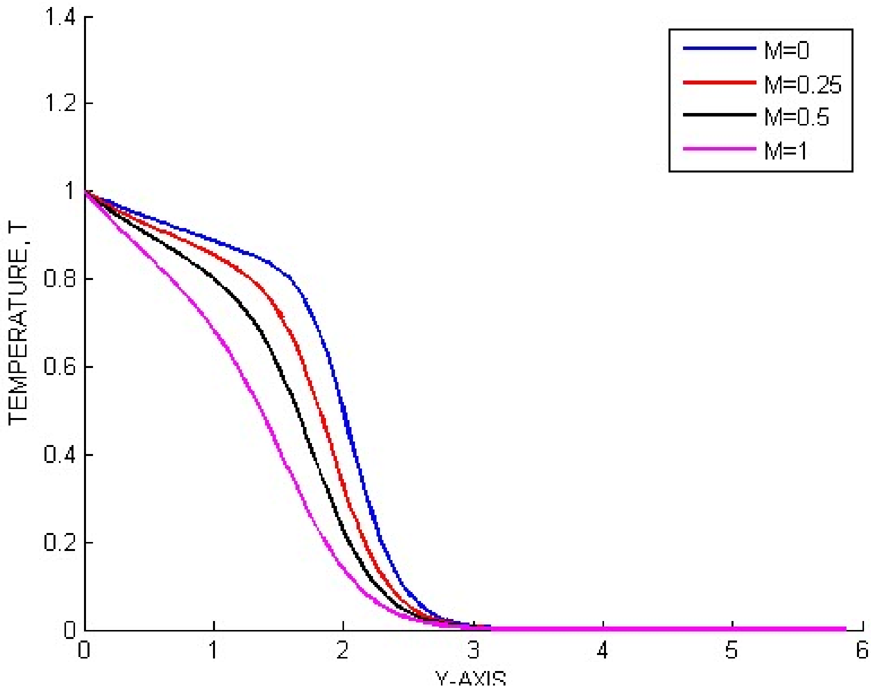

Figure 4.13: Temperature(T)against Y-axis for different values of Magnetic Parameter (M)

Figure 4.13 represents temperature (T) against Y-axis varying Magnetic parameter (M) while $Gr = 85$, $Re = 4$ and $\gamma = 0.5$. It is observed that increase in Magnetic parameter M leads to a decrease in the temperature profiles whereas decrease in magnetic parameter leads to an increase in the temperature profiles. Increase in the Magnetic parameter leads to an increase in the Lorentz force in the fluid which leads to a decrease in the temperature of the fluid and decrease magnetic parameter leads to a decrease in Lorentz force which leads to an increase in the Temperature of the fluid, thus the change in the temperature profiles.

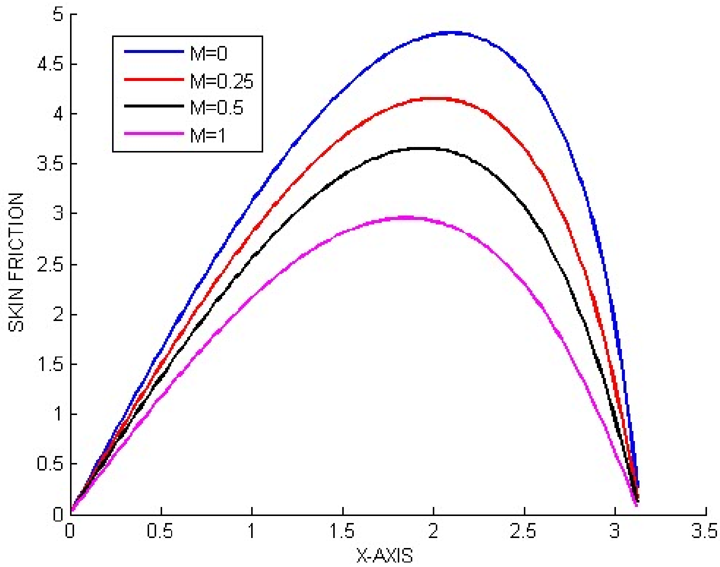

Figure 4.14: Skin Friction against X-axis for different values of Magnetic Parameter (M)

Figure 4.14 above represents Skin friction against X-axis varying Magnetic parameter while $Gr = 85$, $Re = 4$ and $\gamma = 0.5$. It is observed that increase in the Magnetic parameter leads to decrease in the sin friction and decrease in the magnetic parameter leads to an increase in the skin friction of the fluid. This can be explained from the fact that increase in Magnetic parameter leads to increase in Lorentz force which opposes the motion of the fluid and this leads to a decrease in the velocity gradient which leads to decrease in the local skin friction coefficient and hence the reduction in the skin friction profiles.

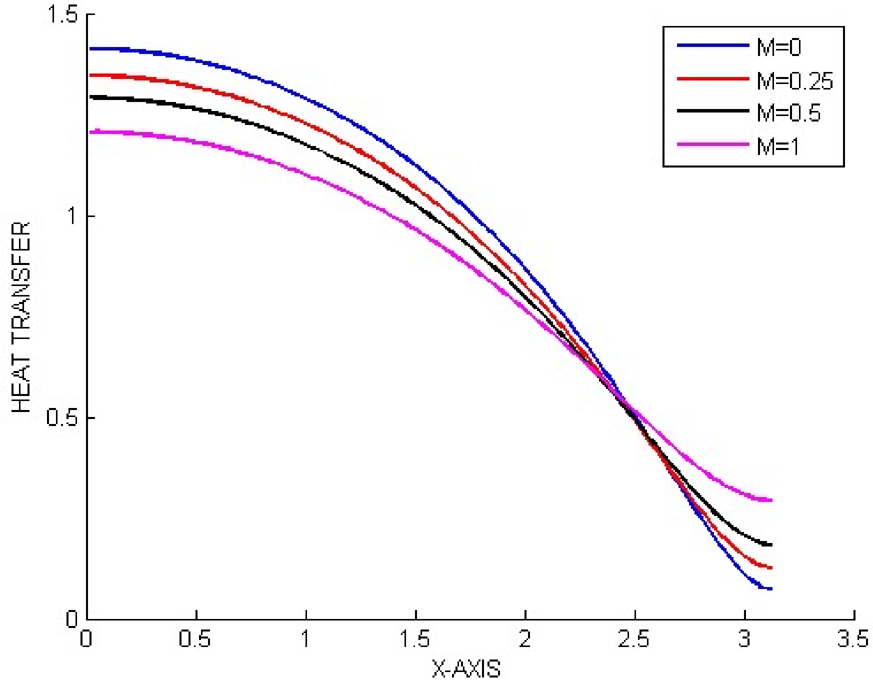

Figure 4.15: Heat Transfer against X-axis for different values of Magnetic Parameter (M)

Figure 4.15 above shows Heat transfer against X-axis varying Magnetic parameter M while $Gr = 85$, $Re = 4$ and $\gamma = 0.5$. It is observed that increase in Magnetic parameter leads to a decrease in the heat transfer and decrease in Magnetic parameter leads to an increase in the heat transfer. This is because increase in Magnetic parameter leads to an increase in Lorentz force which opposes the motion of the fluid and leads to a decrease in the temperature gradient and hence the decrease in the local Nusselt number.

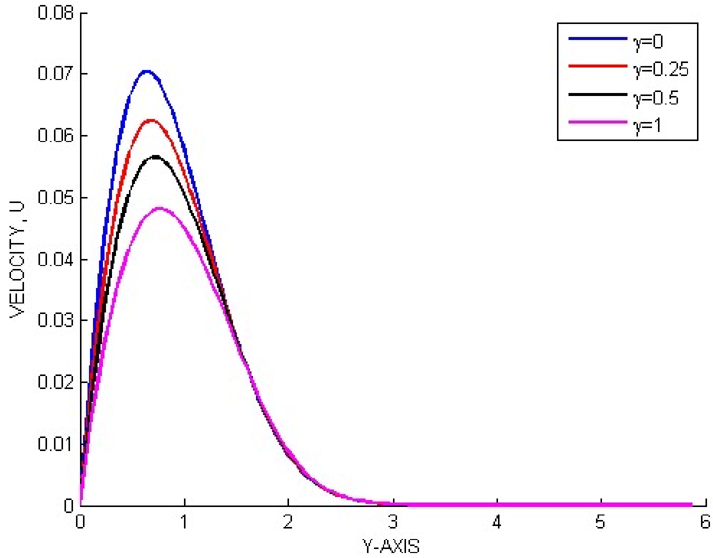

Figure 4.16: Primary Velocity against Y-axis for different values of Viscous Variation Parameter (γ)

Figure 4.16 above shows the primary velocity (U) against Y-axis varying the viscous variation parameter (γ) while $Gr = 85$, $Re = 4$ and $M = 0.5$. From the figure, it is observed that increase in (γ) leads to a decrease in the velocity (U) of the fluid and decrease in (γ) leads to an increase in the velocity of the fluid. For increasing values of (γ) ,the viscosity of the fluid within the boundary layer increases which retards the fluid motion and as a result, the velocity (U) of the fluid decreases.

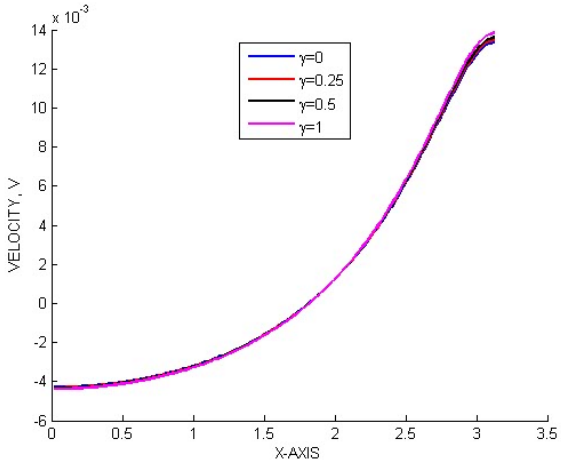

Figure 4.17: Secondary Velocity against X-axis for different values of Viscous Variation Parameter (γ)

Figure 4.17 above represents Secondary velocity (V) against X-axis varying the Viscous variation parameter (γ) while $Gr = 85$, $Re = 4$ and $M = 0.5$. From the figure, it is observed that there is a slight change in the velocity profiles. This is because there is a slight effect of viscosity of the fluid on the secondary velocity of the fluid.

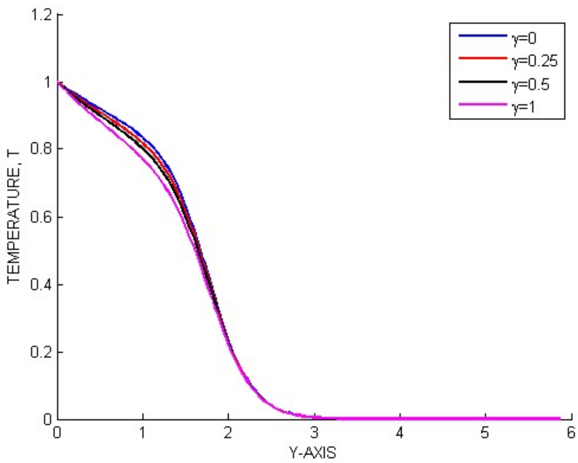

Figure 4.18: Temperature (T) against Y-axis for different values of Viscous Variation Parameter (γ)

Figure 4.18 above represents temperature (T) against Y-axis varying the viscous variation parameter while $Gr = 85$, $Re = 4$ and $M = 0.5$. It is observed that increase in γ leads to a decrease in the temperature of the fluid and decrease in γ leads to an increase in the temperature of the fluid. This is because increase in γ leads to an increase in the viscosity of the fluid which leads to an increase in viscous force which opposes the motion of the fluid. This increase in the viscous force leads to a decrease in the temperature of the flow and the reduction rate of the temperature profiles. The change in the temperature profiles is very slight which shows the change in the viscous variation parameter has a slight effect on the temperature profiles.

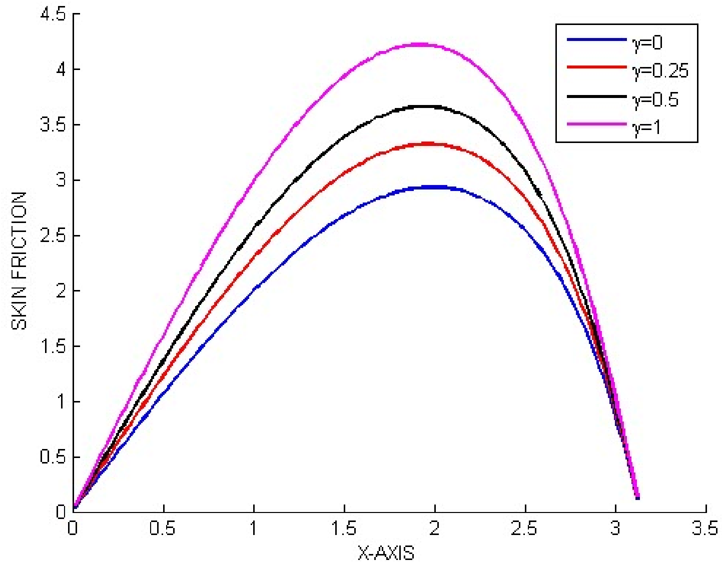

Figure 4.19: Skin Friction against X-axis for different values of Viscous Variation Parameter (γ)

Figure 4.19 above shows Skin friction against X-axis varying the viscous variation parameter while $Gr = 85$, $Re = 4$ and $M = 0.5$. From the figure above, it is observed that increase in viscous variation parameter γ leads to an increase in the skin friction of the fluid and a decrease in γ leads to a decrease in the skin friction of the fluid. Skin friction varies directly proportional to the viscous variation parameter γ and therefore increase in γ leads to an increase in the skin friction and decrease in γ leads to a decrease in the skin friction. Thus, the change in the skin friction profiles as shown above.

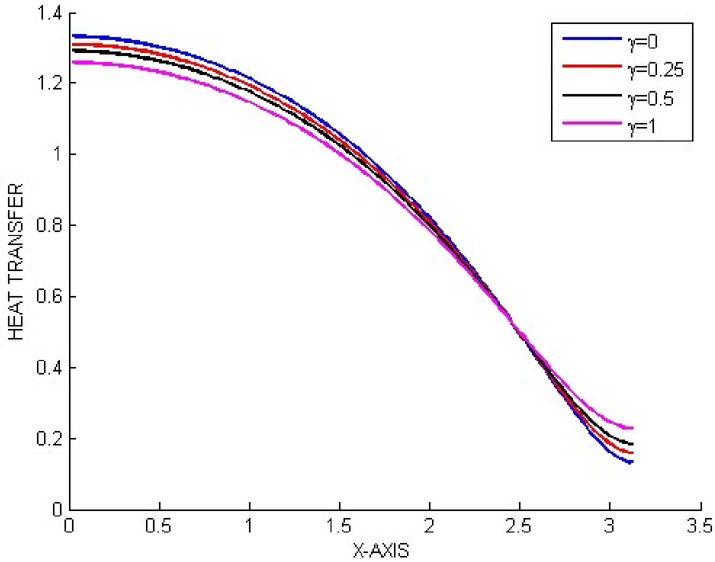

Figure 4.20: Heat Transfer against X-axis for different values of Viscous Variation Parameter (γ)

Figure 4.20 above represents heat transfer against X-axis varying Viscous variation parameter while $Gr = 85$, $Re = 4$ and $M = 0.5$. From the figure, it is observed that increase in the Viscous Variation Parameter leads to a decrease in the heat transfer profiles and decrease in leads to increase in the heat transfer profiles. It can be observed that increase in γ leads to a decrease in the temperature of the fluid and decrease in γ leads to an increase in the temperature of the fluid. This is because increase in γ leads to an increase in the viscosity of the fluid which leads to an increase in viscous force which opposes the motion of the fluid. Decrease in the temperature of the fluid with increase in the viscous variation parameter,γ leads to a decrease in the temperature gradient which leads to a reduction in the rate of heat transfer which means that the Nusselt number decreases.

Chapter 5

CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

This chapter presents the conclusions and recommendations on the areas that require further research.

5.2 Conclusions

In this study, the effects of variable viscosity on unsteady natural convection hydromagnetic flow past an isothermal sphere has been carried out. The numerical results in this study have been obtained using the Direct Numerical Scheme (DNS). It can be concluded that;

i. Increase in Reynolds number (Re) leads to an increase in the secondary velocity (V) and Heat transfer in the fluid but leads to a decrease in Primary velocity (U), Temperature (T) and Skin friction of the fluid.

ii. Increase in Grashof number leads to an increase in Primary velocity (U), Temperature, Skin friction and Heat transfer in the fluid but leads to a decrease in Secondary velocity (V).

iii. Increase in Magnetic parameter (M) leads to an increase in Secondary velocity (V) but leads to a decrease in primary velocity (U),Temperature,(T), Skin Friction and Heat transfer.

iv. Increase in the Viscous variation parameter (γ) leads to an increase in skin friction, a slight change in secondary velocity (V) but leads to a decrease in primary velocity (U), Temperature and Heat transfer in the fluid.

5.3 Recommendations

In this study,effects of variable viscosity on unsteady natural convection hydromagnetic fluid flow past an isothermal sphere taking viscosity as a linear function of temperature has been investigated. Further research can be carried out:

i. When the fluid flow is turbulent.

ii. When the sphere which is immersed in the fluid is non-uniformly heated.

iii. When the fluid is taken to be compressible.

iv. When the sphere is partially immersed in the fluid.

v. When the sphere is porous.

vi. When the sphere is not stationary but it is moving.

vii. When the viscosity is a non-linear function of Temperature.

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1: Computer Code in Matlab

The governing equations (3.77),(3.78),(3.80),(3.81) and (3.82) in matrix notation in finite difference form are simulated in the following computer programme code developed usng MATLAB software subject to the boundary conditions as discussed here in.

The results are obtained by varying various flow parameters notably, Reynolds number, Magnetic number, Grashof number and Viscosity variation parameter.

```
function LucyMwangi()
clear all;clc;
x0=0;xend=pi; dx=pi/180;
x=x0:dx:xend;
nx=floor((xend-x0)/dx);
y0=0;yend=2; dy=0.04;
y=y0:dy:yend;
ny=(yend-y0)/dy;

M=10;Pr=0.73;eta=.015;Gr=4;color='-b';

u=zeros(nx,ny);v=zeros(nx,ny);theta=zeros(nx,ny);

u(1:2,:)=0;v(1:2,:)=0;theta(1:2,:)=0;%BC at X=0
u(nx-1:nx,:)=0;theta(nx-1:nx,:)=1;%BC at X=pi
u(:,1:2)=0;v(:,1:2)=0;theta(:,1:2)=1;%BC at Y=0
u(:,ny-1:ny)=0;theta(:,ny-1:ny)=0;%BC at Y tend to Inf
K=80;
uk=zeros(nx,ny,K);vk=zeros(nx,ny,K);thetak=zeros(nx,ny,K);xk=zeros(nx,ny,K);
matrix=ones(nx,ny,K);SFk=zeros(nx,ny,K);HTk=zeros(nx,ny,K);

for k=1:K
    uk(:,:,1)=u(:,:);vk(:,:,1)=v(:,:);thetak(:,:,1)=theta(:,:);
for i=2:nx-1
    for j=2:ny-1
        xk(i,j,k)=x(i).*matrix(i,j,k);
   vk(i,j,k)=vk(i,j-1,k)-0.5*dy*(uk(i,j-1,k)+uk(i,j,k)).*(1+xk(i,j,k).*(cos(xk(i,j,k))/sin(xk(i,j
```

```
uk(i,j,k)=(dy*dy/(2+M*(dy*dy)))*((1/(dy*dy))*(uk(i,j+1,k)+uk(i-1,j,k))+(eta/(dy*dy)).*(thetak(i,j
      (0.25/(dy*dy))*(thetak(i,j,k)).*(uk(i,j+1,k)+uk(i,j-1,k))+(0.25/(dy*dy))*(thetak(i,j+1,k)-
      (sin(xk(i,j,k))./(xk(i,j,k))).*thetak(i,j,k)-(0.25/dy)*(vk(i,j+1,k)+vk(i-1,j,k)).*(uk(i,j+
      -(0.25/dx)*xk(i,j,k)*(uk(i,j+1,k)+uk(i-1,j,k)).*(uk(i+1,j,k)-uk(i-1,j,k))-(0.25*(uk(i,j+1,

   thetak(i,j,k)=(0.5*Pr*dy*dy)*((1/(Pr*dy*dy))*(thetak(i,j+1,k)+thetak(i,j-1,k))-Gr*(vk(i,j,k))
      -(xk(i,j,k)).*Gr*((uk(i,j,k))).*((thetak(i+1,j,k)-thetak(i-1,j,k))/(2*dx)));

  SFk(i,j,k)=2*(1+eta)*xk(i,j,k).*((uk(i,j+1,k)-uk(i,j-1,k))/(2*dy));
  HTk(i,j,k)=-((thetak(i,j+1,k)-thetak(i,j-1,k))/(2*dy));
   end
end
uk(:,:,k+1)=uk(:,:,k);vk(:,:,k+1)=vk(:,:,k);thetak(:,:,k+1)=thetak(:,:,k);
for ix=1:3
thetak(ix,:,k+1)=thetak(4,:,k+1);%BC at X=0
end
for ix=nx-2:nx
thetak(ix,:,k+1)=thetak(nx-3,:,k+1);%BC at X=pi
end
SFk(:,:,k+1)=SFk(:,:,k);HTk(:,:,k+1)=HTk(:,:,k);
end

figure(1)
mesh(y(2:ny-1),x(2:nx-1),thetak(2:nx-1,2:ny-1,K))
figure(2)
mesh(y(2:ny-1),x(2:nx-1),uk(2:nx-1,2:ny-1,K))
figure(3)
plot(y(1:ny-1),thetak(floor(0.5*nx),1:ny-1,K),color,'LineWidth',2)
xlabel('Y-AXIS')
ylabel('TEMPERATURE')
figure(4)
plot(x(1:nx-1),uk(1:nx-1,floor(0.05*ny),K),color,'LineWidth',2)
```

```
xlabel('Y-AXIS')
ylabel('VELOCITY')
figure(5)
plot(x(2:nx-1),SFk(2:nx-1,2,K),color,'LineWidth',2)
xlabel('X-AXIS')
ylabel('SKIN FRICTION')
figure(6)
plot(x(2:nx-1),HTk(2:nx-1,2,K),color,'LineWidth',2)
xlabel('X-AXIS')
ylabel('HEAT TRANSFER')

end
```

2: Publication

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