

**MARKOV CHAIN ASSET PRICING MODEL FOR  
AN EMERGING MARKET**

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**(Finance Option)**

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**INNOVATION**

**2018**

**MARKOV CHAIN ASSET PRICING MODEL  
FOR AN EMERGING MARKET**

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**MF 300-0002/16**

**A Thesis submitted to Pan African University,  
Institute for Basic Sciences, Technology and  
Innovation in Partial Fulfillment of the Requirements  
for the Degree of Master of Science in Mathematics  
(Finance Option)**

**2018**

# DECLARATION

This thesis is my original work and no part of it has been presented for another degree award in any other university.

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# DEDICATION

To my beloved wife Qaasim Maryam, father Adesokan Abdulrahman and mother Adesokan Fatima.

# ACKNOWLEDGMENTS

I would like to thank my supervisors, Dr. Philip Ngare and Dr. Abdulhakeem Kilishi , for the guidance, and advice they provided throughout the course of this work. I have been extremely fortunate to have Dr. Ngare who responded to my questions and queries so promptly as my supervisor. I would also like to thank my coordinator Prof. George Orwa for his fatherly guidance, and the administrative staff of PAUISTI.

My heartfelt gratitude goes to my wife, for her continued support and encouragement. My mother, father and siblings (Kabeerah, Maryam and Yuusuf) for being more than a family, and my in-laws Mr. and Mrs. Hamzat Omoola, at times they were what kept me going. Also my special teacher and mentor Ustadh Yahya(Aboo ja'far) for his advises and encouragement.

A special mention goes to my academic role model Suleman Nasiru for his help and total support, also to my motivational speaker Mr. Aderibigbe dare, my friends Solihu Abdulgaffar (Fatou's lemma), Adeoye Abeeb (Mr. Amazing) and their families. And to my friend Abdelfattah Mohammed, a brother and confidant my thanks.

Finally, I would like to thank all of my colleagues at PAUISTI and the Pan African Union for their help in my pursuit of this degree, and to everyone everywhere who helped in any way on this journey.

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# ABSTRACT

The chances of success of an investor in the stock market depends heavily on the decisions he takes based on his knowledge of the behavior of the stock market. In this study, the behavior of a stock on the Nigerian stock exchange market was studied. The Markov Chain with a threshold to determine movement between states, was used to estimate expected long and short-run returns, and the result was compared to the expected return of the Capital Asset Pricing Model. It was observed that the mean return of the stock and the expected return of the Capital Asset Pricing Model will not be realized in the long-run regardless of the present state. The study indicates a way to forestall the problem of overpricing or under-pricing returns when using the Capital Asset Pricing Model.

# CHAPTER 1

## INTRODUCTION

### 1.1 Background of the Study

A stock exchange is a well grounded legal framework which facilitates the trading of shares of companies or organizations in a country (Davou et al., 2013). A stock market which is on the rise is a good indicator of the country's economic strengths. An increase in share price is associated with increase in investments and a decrease in share price is linked with decrease in investments.

The dream of every person investing in the stock market is to make profit, however the stock market is a financial market with a high volatility, so the success or failure of the investor heavily depends on the decisions taken which in turn depend on his knowledge of the stock market and strategies or models to predict the movement of prices that could arise as a result of many varying factors.

A number of studies showed that the stock market is affected by macroeconomic variables of the economy with respect to their intensity in different markets. Hence, an investor needs to always be up to date with the behavior of the stock market with the result which is generated after the fluctuation of these variables; an investor needs to know the actions which he has to take and the time he has to make them in order to give him the maximum advantage in making profit while minimizing risks when its elimination is not viable.

Over the years, many models like the Capital Asset Pricing Model(CAPM) by Sharpe (1996) and Lintner (1996), the Geometric Brownian Motion(GBM),Markov chain model

approach used by Zhang and Zhang (2009) to study on forecasting the China's stock market trend etc, have been used to try to predict the behavior of the movement of stock prices, so as to reduce if not eliminate the risk of making losses in the stock market. However there is still a lot of debate as to which model is the most reliable. For example the GBM according to Abidin and Jaffar (2013) can be used to forecast a maximum of two weeks prices, even though their research was carried out on small companies. Also, the GBM fails to account for periods of constant values. The CAPM uses a risk-free rate in determining expected return, but this risk free rate is also susceptible to volatility. All of these shortfalls give fuel to the debate as to which model is the most reliable for making decisions in the stock market.

Hence there is a continuous need to come up with new models or review and upgrade existing models to try as much as is possible to reduce to the barest minimum the chances of failure in a stock market investment.

## **1.2 Problem Statement**

The importance of making well informed decisions that would enhance the chances of success in the stock market can not be over-emphasized. One needs to observe the trend and behaviour of an equity before purchasing stakes in it, as the size of loss which may arise from poor decisions can't be overlooked. Since the stock market is a volatile market which has the random walk property, models which capture volatility would be expected to inform good predictions, as daily stock price records do not conform to usual requirements of constant variance in the common statistical time series.

The conventional CAPM which is used to calculate the expected return on an asset, models either under-valuation or over-valuation of the asset as against the actual returns.

Both cases are a challenge, since an investor might hold off from investing in an asset due to the expectation of the CAPM while it was under-valued and vice-versa.

However the Markov chain unlike the CAPM only gives probabilities and not actual prices of stock and might not be sufficient on its own as an indicator. Also recent study of the stock market using a three-state Markov chain model fails to include a threshold in determining when stock price can be said to have risen, fallen or remained static.

This research used the concept of a threshold to determine when the return of stocks can be said to have risen, fallen, or remained static, which would lead to a change in the transition probabilities, thereafter we analyzed the expected long and short-run return from the Markov chain with the expected return of the conventional CAPM.

### **1.3 Justification of the Study**

There's still an ongoing debate among observers as to which model best covers the description of the stock market behavior . Many models over the years have been proposed, but not without short comings.

For the conventional CAPM, a risk-free rate which is usually the government bonds is needed to calculate the expected return of an asset, also recent study using the Markov chain model have categorized the states into rise or increase, static, fall or decrease while neglecting the use of a threshold to determine when a stock price can be said to have fallen in each of the states mentioned, this will eventually have an effect on the transition probabilities that will be obtained.

Hence there is a need to set a threshold as some movements up or down could actually not reflect a fall in the real sense which will have an effect on the chances of success or failure of investing in a particular asset.

## **1.4 Objectives Of Study**

### **1.4.1 General Objective**

The general objective of this study is to determine asset prices for an emerging market using the Markov chain model.

### **1.4.2 Specific Objectives**

1. To determine a threshold in modeling movement of stock returns in the Markov chain model.
2. To obtain the expected long and short-run returns of the stock Using a three state Markov chain model.
3. To compare the expected long and short-run returns of the Markov chain model with the expected return of the Capital asset pricing model.

## **1.5 Outline of the thesis**

This work contains five chapters which are organized as follows. Chapter 1 gives an introduction to the study, including the problem statement and objectives; Chapter 2 gives the literature review of works on both the Markov chain model and the CAPM; Chapter 3 discusses the methodology used in carrying out the study; Chapter 4 presents the results and discussions and finally, Chapter 5 gives summary, conclusion and recommendations.

# CHAPTER 2

## LITERATURE REVIEW

### 2.1 Introduction

A lot of studies have been carried out to improve decision making in the stock market using a lot of models. Davou et al. (2013); Eseoghene (2011); Maruf and Patrick (2016) all made recent contributions into using the GBM, Markov chain model and CAPM to predict stock price behaviour in the stock market.

### 2.2 Markov Chain Model

Abidin and Jaffar (2013) used the GBM to predict stock price behaviour and found that one week's data is enough to forecast share prices, this seems a positive, but it was also found that the GBM can be used to forecast a maximum of two week closing prices, which is a draw back since investors wont have long term prediction of a stocks price.

In the work of Mills and Jordanov (2003) the returns showed evidence of nonlinearity and non-normality, so conventional auto-correlation analysis was supplemented by the use of a Markov chain technique.

Davou et al. (2013) applied the Markov chain model to model share price movement in the stock market. They discovered that for a company's stock to remain in a static state for long periods is not a good sign of performance. However they failed to consider the long-run prospect of the stock prices as this is the essence of the Markov chain model, its ability to consider volatility for long periods.



Eseoghene (2011) used the Markov chain model to analyze the long-run prospect of stock prices in the stock market, it was found that the model is useful in evaluating long range projections on the future prospects of stock prices. However, the three states employed in modeling the movement of prices were decided without the use of a threshold.

Kilic (2013) used Markov chain methodology to decide whether or not the daily returns of the Istanbul stock exchange (ISE) 100 Indices followed a Martingale (random walk) process. The result for study holds that at any given time, stock prices fully reflect all the available historical information.

Mettle et al. (2014) used Markov chain with finite states for stochastic analysis of share prices. They were able to establish that the states communicate, are aperiodic and ergodic and hence possess limiting distributions. They developed a methodology to determine the expected mean return time for increase in stock price, they posited that decision on investment could be improved if based on high transition probabilities, low mean return time and high limiting distribution. However, like Eseoghene (2011), they did not employ the use of a threshold to determine movement between states, the importance of the threshold can be seen in an instance where (say) price moves from 199.01 to 200.00 and then to 250.05 and then to 200.00, the first movement will be recorded as an increase, even though clearly the second move which is still of higher value than the first will be seen as decrease, and also if decision is based on high transition probabilities, but the steady state distribution for the Fall state is high, it could still lead to a loss in investment.

Maruf and Patrick (2016) also employed a three-state Markov approach to predict asset returns. They failed to assess the market using the probabilities of states (gain to gain and stable to gain against loss to loss and stable to loss) to analyze the future behavior

of the market.

Jasinthan et al. (2015) used the two and three state Markov chain models to analyze the movement of vegetable prices in Jaffna. In the three state Markov chain employed, the concept of a threshold was used, the threshold was determined by the absolute median of the market daily changes in price. The study revealed that the Markov chain models for the daily price movement pattern is same for each vegetable.

Davou et al. (2013) in their paper Markov chain model application on share price movement in stock market used a three state Markov chain, with the state of each day's price depending on the day preceding it without recourse to a threshold.

Kilic (2013) in his work Estimating probability of session returns for Istanbul stock exchange 100 index as Markov chain process, classified returns into eight equal interval of states from high loss (negative return) to positive high return.

Aparna and Sarat (2015) also classified prices into three states without using a threshold, with states of prices depending on successive day's prices. Maruf and Patrick (2016) used a three states Markov chain to predict asset returns and the returns were classified as either positive, zero or negative return values.

Okonta et al. (2017) in their work A Markovian analysis of the Nigerian stock market weekly index, used a two state markov chain, with returns place into states with regards to it being positive or negative.

Jasinthan et al. (2015) used the absolute median of the market daily changes in price as threshold to determine movement between states, stated that there is no unique way to determine threshold in these manner of studies.

## 2.3 Capital asset pricing model

Also for the CAPM which is commonly used to predict stock prices, several studies have been undertaken to improve its result, or to establish its accuracy. It is a widely used model despite its flaws, since it provides an expected value of return as against just probabilities.

According to Graham (2013) CAPM faces three main empirical challenges:

1. The beta anomaly (portfolios of stocks with low beta tend to have higher average returns than the CAPM predicts, while that of stocks with high beta value tend to have lower average returns than the CAPM predicts)
2. the value anomaly (firms with high book-to-market equity [BE/ME] ratios tend to have higher average returns than the firms with low book to market ratios), and
3. the momentum anomaly (stocks with relatively large recent six-month to 12-month returns tend to have higher average returns over the following 12 months than the stocks with relatively low recent six-month to 12-month returns).

Where the beta is a measure of the volatility or systematic risk, of a portfolio in comparison to the market as a whole.

Oke (2013) states that the results of his work generally invalidates the CAPM's predictions that higher risk (beta) is associated with a higher level of return.

Prince and Evans (2013) used the CAPM to predict stock market returns in the Ghana stock exchange, however a constant beta value was used which is not necessarily ideal for best results. It was observed that the stock prices whose expected value was gotten using the CAPM turned out to be either under-priced or over-priced.

Malul et al. (2011) showed how the fundamentals of the CAPM can be utilized to formulate a strategic risk management in a global economy, they considered each country's beta ( $\beta$ ) separately, and found that during a world economic crisis, the loss of growth will be significantly higher in countries with higher betas, and lower in those with lower betas.

## 2.4 Summary

Different studies have been carried out using the Markov chain model to predict movement of prices, Kilic (2013) and Okonta et al. (2017) used the Markov chain model to calculate expected long and short-run returns. Okonta et al. (2017) modeled weekly returns, and found that the Nigerian stock market index will go into a stable state after eight weeks. Prince and Evans (2013) used the CAPM to model expected return of selected stocks on the Ghana stock exchange market, their study showed that all the stocks considered were either over-valued or under-valued.

We compared the Markov chain model's long and short-run expected returns with the expected return of the CAPM over the same stock, to forestall the problem of over-valuation posed by relying only upon the expected return of the CAPM.

# CHAPTER 3

## METHODOLOGY

### 3.1 Introduction

This chapter is in two parts, the first part presents the Markov chain model discussed above, and the method of estimation of a fitting threshold. Modeling random movement of events is a very important aspect of stochastic analysis. Random movement of events if well studied can be beneficial to businesses, investors, among others.

The Markov chain model named after Andrey Markov, who produced the first theoretical results in 1906. A generalization to countably infinite state spaces as produced by Kolmogorov in 1936. The second part presents the Capital Asset Pricing Model.

### 3.2 Markov Process

A stochastic process  $X(t)$  is called a first order Markov process if for any  $t_0 < t_1 < \dots < t_n$ ,

$$P[X(t_n) \leq x_n | X(t_{n-1}) = x_{n-1}, X(t_{n-2}) = x_{n-2}, \dots, X(t_0) = x_0] = P[X(t_n) \leq x_n | X(t_{n-1}) = x_{n-1}]$$

that is, given the present state of the process, the future state is independent of the past.

This property is known as the Markov property.

We note that for Second-order Markov process, the future state depends on both the current state and the immediate past state, and so on for Markov processes of higher order.

Markov processes are classified according to the nature of the time parameter and the nature of the state space. With regards to the state space, a Markov process can either be a discrete-state Markov process or a continuous-state Markov process. A discrete-state Markov process is called a Markov chain.

Similarly, with respect to time, a Markov process can be either a discrete-time Markov process or a continuous-time Markov process. Thus, Markov processes are basically of four types.

1. Discrete-time Markov chain (discrete-time discrete-state Markov process)
2. Continuous-time Markov chain (continuous-time discrete-state Markov process)
3. Discrete-time Markov process (discrete-time continuous-state Markov process)
4. Continuous-time Markov process (continuous-time continuous-state Markov process)

This work is based on a discrete time and discrete state space (Discrete-time Markov chain), and we refer to it as simply Markov chain.

### 3.3 Markov chain model

The discrete-time process  $X_k, k = 0, 1, 2, \dots$  is a Markov chain if  $\forall i, j, k, \dots, m$ , we have

$$P[X_k = j | X_{k-1} = i, X_{k-2} = n, \dots, X_0 = m] = P[X_k = j | X_{k-1} = i] = p_{ijk} \quad (3.1)$$

where  $p_{ijk}$  is called the state-transition probability, which is the conditional probability that the process will be in state  $j$  at time  $k$  immediately after the next transition, given that it is in state  $i$  at time  $k-1$ . A Markov chain that follows the preceding rule is called

a *nonhomogenous* Markov chain. This work uses the *homogenous* Markov chain, that is a Markov chain, that is independent of time. And we have

$$P[X_k = j | X_{k-1} = i, X_{k-2} = n, \dots, X_0 = m] = P[X_k = j | P[X_{k-1} = i]] = p_{ij} \quad (3.2)$$

**Theorem 3.1.**  $(X_n)_{0 \leq n \leq N}$  is Markov if and only if

$$P(X_0 = i_0, X_1 = i_1, \dots, X_N = i_N) = \lambda_{i_0} p_{i_0 i_1} p_{i_1 i_2} \cdots p_{i_{N-1} i_N}. \quad (3.3)$$

**Proof.** First, suppose  $(X_n)_{0 \leq n \leq N}$  is Markov, thus

$$\begin{aligned} & P(X_0 = i_0, X_1 = i_1, \dots, X_N = i_N) \\ &= P(X_0 = i_0) P(X_1 = i_1 | X_0 = i_0) \cdots P(X_N = i_N | X_0 = i_0, \dots, X_{N-1} = i_{N-1}) \\ &= P(X_0 = i_0) P(X_1 = i_1 | X_0 = i_0) \cdots P(X_N = i_N | X_{N-1} = i_{N-1}) \\ &= \lambda_{i_0} p_{i_0 i_1} p_{i_1 i_2} \cdots p_{i_{N-1} i_N} \end{aligned}$$

Now assume that (3.3) holds for N, thus

$$\begin{aligned} & P(X_0 = i_0, X_1 = i_1, \dots, X_N = i_N) = \lambda_{i_0} p_{i_0 i_1} p_{i_1 i_2} \cdots p_{i_{N-1} i_N} \\ & \sum_{i_N \in I} P(X_0 = i_0, X_1 = i_1, \dots, X_N = i_N) = \sum_{i_N \in I} \lambda_{i_0} p_{i_0 i_1} p_{i_1 i_2} \cdots p_{i_{N-1} i_N} \\ & P(X_0 = i_0, X_1 = i_1, \dots, X_{N-1} = i_{N-1}) = \lambda_{i_0} p_{i_0 i_1} p_{i_1 i_2} \cdots p_{i_{N-2} i_{N-1}} \end{aligned}$$

And now by induction, (3.3) holds for all  $0 \leq n \leq N$ . From the formula for conditional

probability, namely that  $P(A|B) = P(A \cap B)/P(B)$ , we can show that

$$\begin{aligned} P(X_{N+1} = i_{N+1} | X_0 = i_0, \dots, X_N = i_N) &= \frac{P(X_0 = i_0, \dots, X_N = i_N, X_{N+1} = i_{N+1})}{P(X_0 = i_0, \dots, X_N = i_N)} \\ &= \frac{\lambda_{i_0} p_{i_0 i_1} \cdots p_{i_{N-1} i_N} p_{i_N i_{N+1}}}{\lambda_{i_0} p_{i_0 i_1} \cdots p_{i_{N-1} i_N}} \\ &= p_{i_N i_{N+1}} \end{aligned}$$

Thus, by definition,  $(X_n)_{0 \leq n \leq N}$  is Markov( $\lambda, P$ ).

### 3.3.1 Classification of states

A state  $j$  is said to be accessible from state  $i$ , if starting from  $i$ , it is possible that the process will ever enter state  $j$ , that is  $p_{ij} > 0$ .

Two states  $i$  and  $j$  are said to communicate, if  $i$  and  $j$  are accessible from each other, and are said to be in the same class. A Markov chain in which all states communicate is called irreducible.

### 3.3.2 Communication Classes and Recurrence

State  $j$  is accessible from state  $i$ , and we write  $i \rightarrow j$  if

$$P_i(X_n = j \text{ for some } n \geq 0) > 0.$$

Also  $i$  communicates with  $j$ , and we write  $i \leftrightarrow j$  if both  $i \rightarrow j$  and  $j \rightarrow i$ .

**Theorem 3.2.** For distinct states  $i, j \in I$ ,  $i \rightarrow j \iff p_{i i_1} p_{i_1 i_2} \cdots p_{i_{n-1} j} > 0$  for some states  $i_1, i_2, \dots, i_{n-1}$ . Also,  $\leftrightarrow$  is an equivalence relation on  $I$ .



**Proof.** ( $\Rightarrow$ )

$$0 < P_i(X_n = j \text{ for some } n \geq 0) \leq \sum_{n=0}^{\infty} P_i(X_n = j) = \sum_{n=0}^{\infty} \sum_{i_1, \dots, i_{n-1}} p_{ii_1} p_{i_1 i_2} \cdots p_{i_{n-1} j}$$

Thus, for some  $p_{ii_1} p_{i_1 i_2} \cdots p_{i_{n-1} j} > 0$  for some states  $i_1, i_2, \dots, i_{n-1}$ .

( $\Leftarrow$ ) Take some  $i_1, i_2, \dots, i_{n-1}$  such that

$$0 < p_{ii_1} p_{i_1 i_2} \cdots p_{i_{n-1} j} \leq P_i(X_n = j) \leq P_i(X_n = j \text{ for some } n \geq 0).$$

Now it is clear from the proven inequality that  $i \rightarrow j, j \rightarrow k \Rightarrow i \rightarrow k$ . Also, it is true that  $i \leftrightarrow i$  for any state  $i$  and that  $i \leftrightarrow j \Rightarrow j \leftrightarrow i$ . Thus,  $\leftrightarrow$  is an equivalence relation on  $I$ .

We say that  $\leftrightarrow$  partitions  $I$  into *communication classes*. Also, a Markov chain or transition matrix  $P$  where  $I$  is a single communication class is called *irreducible*.

**Theorem 3.3.** Let  $C$  be a communication class. Either all states in  $C$  are recurrent or all are transient.

**Proof.** Take any distinct pair of states  $i, j \in C$  and suppose that  $i$  is transient. Then there exist  $n, m \geq 0$  such that  $P_i(X_n = j) > 0$  and  $P_j(X_m = i) > 0$ , and for all  $r \geq 0$

$$P_i(X_{n+r+m} = i) \geq P_i(X_n = j) P_j(X_r = j) P_j(X_m = i).$$

This implies that

$$\sum_{r=0}^{\infty} P_j(X_r = j) \leq \frac{1}{P_i(X_n = j) P_j(X_m = i)} \sum_{r=0}^{\infty} P_i(X_{n+r+m} = i) < \infty$$

So any arbitrary  $j$  is transient, so the whole of  $C$  is transient. The only way for this not to be true is if all states in  $C$  are recurrent.

This theorem shows us that recurrence and transience is a class property.

**Theorem 3.4.** Suppose  $P$  is irreducible and recurrent. Then for all  $i \in I$  we have  $P(T_i < \infty) = 1$ .

**Proof.** By Theorem 3.2 we have

$$P(T_i < \infty) = \sum_{j \in I} P_j(T_i < \infty) P(X_0 = j)$$

so we only need to show  $P_j(T_i < \infty) = 1$  for all  $j \in I$ . By the irreducibility of  $P$ , we can pick an  $m$  such that  $P_i(X_m = j) > 0$ . we have

$$\begin{aligned} 1 &= P_i(X_n = i \text{ for infinitely many } n) \\ &= P_i(X_n = i \text{ for some } n \geq m + 1) \\ &= \sum_{k \in I} P_i(X_n = i \text{ for some } n \geq m + 1 \mid X_m = k) P_i(X_m = k) \\ &= \sum_{k \in I} P_k(T_i < \infty) P_i(X_m = k) \end{aligned}$$

using Theorem 3.2 again. Since  $\sum_{k \in I} P_i(X_m = k) = 1$  so we have that

$$P_j(T_i < \infty) = 1.$$

### 3.3.3 Stopping Times and the Strong Markov Property

We start this section with the definition of a stopping time. A random variable  $T$  is called a *stopping time* if the event  $\{T = n\}$  depends only on  $X_0, \dots, X_n$  for  $n = 0, 1, 2, \dots$

An example of a stopping time would be the *first passage time*

$$T_i = \inf\{n \geq 1 \mid X_n = i\}.$$

where we define  $\inf \emptyset = \infty$ . This is a stopping time since  $\{T_i = n\} = \{X_k \neq i, X_n = i \mid 0 < k < n\}$ . Now we will define an expansion of this idea that we will use later.

The *rth passage time*  $T_i^{(r)}$  to state  $i$  is defined recursively using the first passage time.

$$T_i^{(0)} = 0, \quad T_i^{(1)} = T_i$$

and, for  $r = 1, 2, \dots$ ,

$$T_i^{(r+1)} = \inf\{n \geq T_i^{(r)} + 1 \mid X_n = i\}.$$

This leads to the natural definition of the *length of the rth excursion to i* as

$$S_i^{(r)} = \begin{cases} T_i^{(r)} - T_i^{(r-1)} & \text{if } T_i^{(r-1)} < \infty \\ 0 & \text{otherwise.} \end{cases}$$

The following theorem shows how the Markov property holds at stopping times.

**Theorem 3.5.** Let  $T$  be a stopping time of  $(X_n)_{n \geq 0}$  which is Markov( $P$ ). Then given  $T < \infty$  and  $X_T = i$ ,  $(X_l)_{l \geq T}$  is Markov( $\delta_i, P$ ) and independent of  $X_k$ ,  $0 \leq k < T$ .

**Proof.** First, we already have that  $(X_l)_{l \geq T}$  is Markov( $\delta_i, P$ ) by Theorem 1.4, so we just need to show the independence condition. Let the event  $A = \{X_T = i_0, \dots, X_{T+n} = i_n\}$  and the event  $B$  be any event determined by  $X_0, \dots, X_T$ . It is important to notice that

the event  $B \cap \{T = m\}$  is determined by  $X_0, \dots, X_m$ . We get that

$$P(A \cap B \cap \{T = m\} \cap \{X_T = i\}) = P_i(X_0 = i_0, \dots, X_n = i_n)P(B \cap \{T = m\} \cap \{X_T = i\})$$

If we now sum over  $m = 0, 1, 2, \dots$  and divide each side by  $P(T < \infty, X_T = i)$  using the definition of conditional probability, we obtain

$$P(A \cap B | T < \infty, X_T = i) = P_i(X_0 = i_0, \dots, X_n = i_n)P(B | T < \infty, X_T = i)$$

which gives us the independence we desired.

### 3.3.4 Limiting-state probabilities

The n-step state-transition probability  $p_{ij}(n)$  is the conditional probability that the system will be in state  $j$  after exactly  $n$  transitions, given that it is presently in state  $i$ .

The n-step transition probability can be obtained by multiplying the transition probability matrix by itself  $n$  times.

For the class of Markov chains in which the limit exists, we define the limiting-state probabilities as:

$$\lim_{n \rightarrow \infty} P[X(n) = j] = \pi_j; n = 1, 2, \dots, N$$

since the n-step transition probability can be written in the form

$$p_{ij}(n) = \sum_k p_{ik}(n-1)p_{kj}$$

So if the limiting-state probabilities exist and do not depend on the initial state, we have

$$\lim_{n \rightarrow \infty} P[X(n) = j] = \pi_j = \lim_{n \rightarrow \infty} \sum_k p_{ik}(n-1)p_{kj} = \sum_k \pi_k p_j$$

defining the limiting state probability vector as  $\pi = [\pi_1, \pi_2, \dots, \pi_n]$ , then we have

$$\pi_j = \sum_k \pi_k p_j$$

$$\pi = \pi P$$

$$1 = \sum_j \pi_j$$

### 3.3.5 Assumptions of a Markov chain model

For any process to be modeled by the Markov chain model, the following assumptions must hold:

1. The present state of the process will depend only on the immediate past state.
2. Transition probability matrices are the result of processes that are stationary in time or space; the transition probability does not change with time or space

**Definition 3.1.1** A Markov chain is collection of random variables  $X_t$  (where  $t = 0, 1, \dots$ ) having the property that, given the present, the future is conditionally independent of the past. In other words,

$$P(X_i = j | X_0 = i_0, X_1 = i_1, \dots, X_{t-1} = i_{t-1}) = P(X_t = j | X_{t-1} = i_{t-1}) \quad (3.4)$$

**Definition 3.1.2** The *one-step transition probability* is the probability of transitioning

from one state to another in a single step. The Markov chain is said to be time homogeneous if the transition probabilities from one state to another are independent of time index.

$$P_{ij} = P_r(X_n = j | X_{n-1} = i) \quad (3.5)$$

The *transition probability matrix*,  $P$ , is the matrix consisting of the one-step transition probabilities.

The *m-step transition probability* is the probability of transitioning from state  $i$  to state  $j$  in  $m$  steps.

$$P_{ij}^{(m)} = P_r(X_{n+m} = j | X_n = i) \quad (3.6)$$

The  $m$ -step transition matrix whose elements are the  $m$ -step transition probabilities is denoted by  $P^{(m)}$

**Definition 3.1.3** If for any sequence of distribution  $\pi_n$  where  $n = 0, 1, \dots, \lim_{n \rightarrow \infty} \pi^n = \pi$  and  $\sum \pi = 1$  then  $\pi$  is called the limiting distribution of the chain.

**Definition 3.1.4** If at any step  $n$ ,  $P^{(n)} * \pi = \pi$  then it implies that the chain has reached the steady state or equilibrium and  $\pi$  is called the steady state distribution

### 3.3.6 Estimating Transition probabilities

The maximum likelihood method will be applied to estimate the transition probabilities  
Consider

$$P_{ij} = P_r(X_{t+1} = j | X_t = i) \quad (3.7)$$

Where  $x_i \leq x_1, x_2, \dots, x_n$  is a realization of the random variable  $X_1^n$ , and the probability of this realization is

$$\begin{aligned}
P_r(X_1^n = x_1^n) &= P_r(X_1 = x_1) \prod_{t=2}^n P_r(X_t = x_t | X_1^{t-1} = x_1^{t-1}) \\
&= P_r(X_1 = x_1) \prod_{t=2}^n P_r(X_t = x_t | X_1^{t-1} = x_1^{t-1})
\end{aligned} \tag{3.8}$$

Which can be rewritten in terms of the transition probabilities  $P_{ij}$  to get the likelihood of a given transition matrix

$$L(p) = P_r(X_1 = x_1) \prod_{t=2}^n P_{(x_{t-1})(x_t)} \tag{3.9}$$

Let  $n_{ij} \equiv$  number of times  $i$  is followed by  $j$  in  $X_1^n$  and re-write the likelihood in terms of them

$$L(p) = P_r(X_1 = x_1) \prod_{i=1}^k \prod_{j=1}^k P_{ij}^{n_{ij}} \tag{3.10}$$

Maximizing the likelihood with respect to  $P_{ij}$ , the logarithm of (3.7) is first taken. Hence,

$$\begin{aligned}
\iota(p) &= \log L(p) = \log P_r(X_1 = x_1) + \sum_{i,j} n_{ij} \log P_{ij} \\
&\Rightarrow \frac{\partial \iota}{\partial P_{ij}} = \frac{n_{ij}}{P_{ij}}
\end{aligned} \tag{3.11}$$

Equating the first derivative of (3.8) with respect to  $P_{ij}$  to zero yields

$$\frac{n_{ij}}{P_{ij}} = 0$$

Considering  $\sum_j P_{ij} = 1$  and picking one of the transition probabilities to express in terms of the others, say it's the probability of going to 1, so for each  $i$ ,  $P_{i1} = 1 - \sum_{j=2}^n P_{ij}$ .

Now, take derivatives of the likelihood and have  $\frac{\partial}{\partial P_{i1}}$

$$\frac{\partial \ell}{\partial P_{ij}} = \frac{n_{ij}}{P_{ij}} - \frac{n_{i1}}{P_{i1}} \quad (3.12)$$

setting this equal to zero at the MLE  $\hat{P}$ ,

$$\frac{n_{ij}}{\hat{P}_{ij}} = \frac{n_{i1}}{\hat{P}_{i1}} \quad (3.13)$$

$$\frac{n_{ij}}{n_{i1}} = \frac{\hat{P}_{ij}}{\hat{P}_{i1}} \quad (3.14)$$

since this holds  $\forall j \neq 1$ , we conclude  $\hat{P}_{ij} \propto n_{ij}$  and infact

$$P_{ij} = \frac{n_{ij}}{\sum_j n_{ij}} \quad (3.15)$$

**Proposition 3.1.** Chapman-Kolmogorov equation  $\forall 0 < r < n$

$$p_{ij}(n) = \sum_k p_{ik}(r)p_{kj}(n-r)$$

That is, the probability that the process starts in a state  $i$  and goes to state  $j$  at the end of the  $n$ th transition is the product of the probability that the process starts in state  $i$  and goes to an intermediate state  $k$  after  $r$  transitions and the probability that it goes from state  $k$  to state  $j$  after additional  $n-r$  transitions.



**Proof.**

$$\begin{aligned}
p_{ij} &= P[X_n = j | X_0 = i] \\
&= \sum_k P[X_n = j, X_r = k | X_0 = i] \\
&= \sum_k P[X_n = j | X_r = k, X_0 = i] P[X_r = k | X_0 = i] \\
&= \sum_k P[X_n = j | X_r = k] P[X_r = k | X_0 = i] \\
&= \sum_k p_{kj}(n-r) p_{ik}(r) = \sum_k p_{ik}(r) p_{kj}(n-r)
\end{aligned} \tag{3.16}$$

### 3.3.7 Estimating Threshold

This work uses the concept of quartiles as threshold to determine movement between the states of the Markov chain. Quartiles (say;  $Q_1, Q_2, Q_3,$ ) are values that divide data into four quarters, there are 25% of data both sides of  $Q_1$ ,  $Q_2$  represents the median of the data, and  $Q_3$  is the third quartile value also having 25% of data both sides of it.

### 3.3.8 Estimating Expected Returns

The expected long and short-run returns were calculated using the formula given below (Kilic 2013). With long-run as

$$E_R = \pi_j \mu_i \tag{3.17}$$

While that of short-run as

$$E_r = P^n \mu_i \tag{3.18}$$

where  $E_R$  and  $E_r$  are the expected long and short-run returns respectively,  $\pi_j$  is the steady-state probability,  $P^n$  is the limiting probability and  $\mu_i$  is the mean returns of

state  $i$ .

### 3.3.9 Application to data

We intend to use data from the Nigerian stock exchange.

We classify each day's return into three states depending on its place in the division of the data using quartiles. Hence we have:

**First state (rise):** Here, the return is greater than the value of the third quartile  $Q_3$ .

**Second state (stable):** Here today's return is within or equal to the value of the first and third quartiles  $Q_1$  and  $Q_3$ .

**Third State (fall):** Here, the return is less than the value of the first quartile  $Q_1$ .

Let  $Y_n$  denote the closing day's price of a stock in the stock market on the  $n^{\text{th}}$  day, and  $Y_{n-1}$  denote the closing day's price of the previous day, then we define a random variable  $Z_n$  as

$$Z_n = \frac{Y_n - Y_{n-1}}{Y_{n-1}} \quad (3.19)$$

The above states can be represented as a trinary random variable  $X_n$  denoted by

$$X_n = \begin{cases} 1, & \text{if } Z_n > Q_3 \\ 2, & \text{if } Q_1 \leq Z_n \leq Q_3 \\ 3, & \text{if } Z_n < Q_1 \end{cases} \quad (3.20)$$

Hence we say the random variable  $X_n$  is a Markov chain of three states with state space 1, 2, 3. Where  $Q_1$  and  $Q_3$  are the first and third quartiles of the data set. So we get a transition frequency of the form where  $n_{ij}(i, j = 1, 2, 3)$  represents the number of times transition is made from state  $i$  to state  $j$ , and  $S_i(i = 1, 2, 3)$  represents the sum of values

Table 3.1: **Transition Frequency**

State	Rise	Stable	Fall	Sum
Rise	$n_{11}$	$n_{12}$	$n_{13}$	$S_1$
Stable	$n_{21}$	$n_{22}$	$n_{23}$	$S_2$
Fall	$n_{31}$	$n_{32}$	$n_{33}$	$S_3$

in each row  $i$ .

We get the transition matrix as shown in (3.1.4),

$$P_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}$$

which is of the form

$P =$

$$\begin{matrix} & \begin{matrix} Fall & Rise & Stable \end{matrix} \\ \begin{pmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{pmatrix} \end{matrix}$$

where  $P_{ij}$  represents the probability of moving from state  $i$  to state  $j$ .

### 3.4 Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM) is a simple yet powerful tool used by investors to determine the risk and reward of a stock, the model basically tells us how much compensation is to be expected for taking risks.

The CAPM can be divided basically into two parts, the first part takes into consideration the time value of money, this is represented in the formula by the risk-free rate, this is

like a compensation for investing in a stock over a period of time, that is; the return an investor is willing to accept on an investment without risk. The yield on government bonds are usually used to depict the risk-free rate.

The second part of the CAPM formula considers risk and calculates the return an investor is required to take as compensation for taking on additional risk, since there's no reason to carry more risks if returns remain the same as or lower than the risk-free rate.

The CAPM formula is given as follows

$$E_r = r_f + \beta(r_m - r_f) \quad (3.21)$$

where

$E_r$  is the expected return on the stock

$r_f$  is the risk-free rate

$r_m$  is the expected market return and

$\beta$  is the beta of the stock.

The  $\beta$  (beta) serves as a risk measure, it reflects how risky an asset is compared to overall risk of the market. So the higher the  $\beta$  of a stock, the riskier it is and hence the higher the return the investor expects. The relation  $(r_m - r_f)$  which is the difference between the expected market return and the risk free rate is also referred to as the Market risk premium.

### 3.4.1 Application of the CAPM

The risk premium of the CAPM was gotten from the relation  $(r_m - r_f)$  where  $r_m$  will be the average return of the market for the year and  $r_f$  the average risk-free rate of return of

the year, which is the yield on the government treasury bill which is relatively risk-free. The intention is to calculate the beta, which is a measure of the stock's volatility over time in relation to a market benchmark, using the Microsoft excel software, as against using online calculators, this gives us full control and understanding of the stock. So we calculate the  $\beta$  for our stock using the closing prices of the stock, and the chosen benchmark. We first calculate the daily percentage change for the stock and the index, using

$$D\%C = ((Y_n - Y_{n-1})/Y_{n-1}) * 100 \quad (3.22)$$

where

$Y_n$  denotes today's closing price, and

$Y_{n-1}$  denotes yesterday's closing price.

then, we compute the  $\beta$  using the formula

$$\beta = \frac{Cov(stock's\%dailychange, index's\%dailychange)}{Var(index's\%dailychange)} \quad (3.23)$$

# CHAPTER 4

## DATA ANALYSIS AND RESULTS

The daily closing prices of Nestle Nigeria PLC from the Nigerian stock exchange was used in this study. The data was obtained from Financial Times website [www.ft.com](http://www.ft.com)

The daily returns were classified into three states depending on where its value falls in the quartile division.

Let  $Y_n$  denote the closing day's price of the stock on the  $n^{th}$  day, and  $Y_{n-1}$  denote the closing day's price of the previous day. The returns  $Z_n$  were estimated using equation (3.19).

The states were represented as a trinary random variable  $X_n$  as shown in equation (3.20) Hence we say the random variable  $X_n$  is a Markov chain of three states with state space 1,2,3. Where  $Q_1$  and  $Q_3$  are the first and third quartiles of the data set. So we get a transition frequency as shown in table (3.1). The transition matrix is obtained using equation (3.15).

For the CAPM, the beta which is a measure of the stock's volatility over time in relation to a market benchmark was estimated using the Microsoft excel software, as against using online calculators, this gives us full control and understanding of the stock. The  $\beta$  for the stock was estimated using the closing prices of the stock, and the chosen benchmark. The daily percentage change for the stock and the index was obtained using equation (3.21), then we compute the  $\beta$  using equation (3.22).

### 4.0.1 Markov Chain Model

The successive day's prices were used to calculate the return, using  $Z_n = \frac{Y_n - Y_{n-1}}{Y_{n-1}}$  where  $Y_n$  and  $Y_{n-1}$  represents today's and yesterday's closing price respectively, and  $Z_n$  the returns. The values of the quartiles  $Q_1$  and  $Q_3$  were gotten to be  $-0.0005168$  and  $0.0076744$  respectively, the mean of the returns  $Z_n$  was gotten as  $0.0013074$ . Also, we get  $\mu$  the mean return of each state as  $\mu = [-0.0317283, 0.0357917, 0.0003020]$ .

Table 4.1: **Descriptive statistics of Returns**

Sample size	237
Minimum	-0.0973746
Q1	-0.0005168
Median	-0.0000000
Q3	0.0076744
Mean	0.0013074
Maximum	0.0592789

So we got the transition frequency below.

Table 4.2: **Transition Frequency**

State	Fall	Rise	Stable	Sum
Fall	17	16	26	59
Rise	18	23	19	60
Stable	25	21	72	118

We get the transition matrix of the form





$$P^1 = \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{ccc} \textit{Fall} & \textit{Rise} & \textit{Stable} \\ \left( \begin{array}{ccc} 0.2881356 & 0.2711864 & 0.4406780 \\ 0.3000000 & 0.3833333 & 0.3166667 \\ 0.2118644 & 0.1779661 & 0.6101695 \end{array} \right) \end{array}$$

$$P^2 = \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{ccc} \textit{Fall} & \textit{Rise} & \textit{Stable} \\ \left( \begin{array}{ccc} 0.2577420 & 0.2605190 & 0.4817390 \\ 0.2685311 & 0.2846563 & 0.4468127 \\ 0.2437087 & 0.2342646 & 0.5220267 \end{array} \right) \end{array}$$

$$P^3 = \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{ccc} \textit{Fall} & \textit{Rise} & \textit{Stable} \\ \left( \begin{array}{ccc} 0.2544837 & 0.2554949 & 0.4900214 \\ 0.2574339 & 0.2614577 & 0.4811084 \\ 0.2510994 & 0.2487950 & 0.5001056 \end{array} \right) \end{array}$$

$$P^4 = \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{ccc} \textit{Fall} & \textit{Rise} & \textit{Stable} \\ \left( \begin{array}{ccc} 0.2537924 & 0.2541594 & 0.4920482 \\ 0.2545429 & 0.2556590 & 0.4897981 \\ 0.2529437 & 0.2524680 & 0.4945883 \end{array} \right) \end{array}$$

⋮

$$P^{12} = \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{ccc} \textit{Fall} & \textit{Rise} & \textit{Stable} \\ \left( \begin{array}{ccc} 0.2535647 & 0.2537065 & 0.4927289 \\ 0.2535647 & 0.2537065 & 0.4927288 \\ 0.2535646 & 0.2537064 & 0.4927289 \end{array} \right) \end{array}$$

$$P^{13} = \begin{matrix} & \begin{matrix} Fall & Rise & Stable \end{matrix} \\ \begin{pmatrix} 0.2535647 & 0.2537065 & 0.4927289 \\ 0.2535647 & 0.2537065 & 0.4927289 \\ 0.2535646 & 0.2537065 & 0.4927289 \end{pmatrix} \end{matrix}$$

$$P^{14} = \begin{matrix} & \begin{matrix} Fall & Rise & Stable \end{matrix} \\ \begin{pmatrix} 0.2535647 & 0.2537065 & 0.4927289 \\ 0.2535647 & 0.2537065 & 0.4927289 \\ 0.2535647 & 0.2537065 & 0.4927289 \end{pmatrix} \end{matrix}$$

Figure 4.1 is the transition diagram, representing transition between the three states.

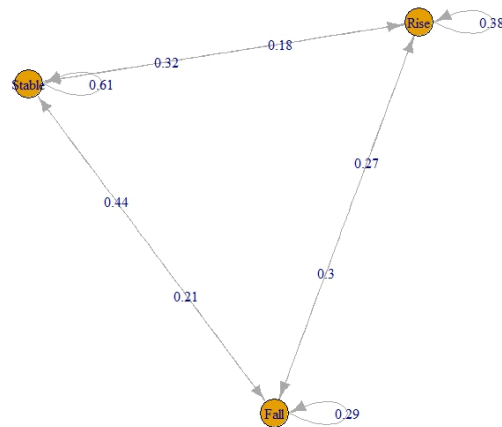


Figure 4.1: **Transition diagram**

## 4.0.2 Expected Returns

The expected long and short-run returns were calculated using equations (3.12) and (3.13) respectively. For the expected long-run return we multiply the steady state probability

vector with the mean returns of states, i.e

$$E_R = \pi_j \mu_i$$

$$\begin{bmatrix} 0.2535647 & 0.2537065 & 0.4927289 \end{bmatrix} \begin{bmatrix} -0.0317283 \\ 0.0357917 \\ 0.0003020 \end{bmatrix} = 0.001184214$$

And for expected short-runs, we have

$$E_r = P^j * \mu_i$$

for  $j = 1, 2, 3, \dots, n$

Now, for  $j = 1$

$$\begin{bmatrix} 0.2881356 & 0.2711864 & 0.4406780 \\ 0.3000000 & 0.3833333 & 0.3166667 \\ 0.2118644 & 0.1779661 & 0.6101695 \end{bmatrix} \begin{bmatrix} -0.0317283 \\ 0.0357917 \\ 0.0003020 \end{bmatrix} = \begin{bmatrix} 0.0006972543 \\ 0.0042972938 \\ -0.0001681168 \end{bmatrix}$$

for  $j = 2$

$$\begin{bmatrix} 0.2577420 & 0.2605190 & 0.4817390 \\ 0.2685311 & 0.2846563 & 0.4468127 \\ 0.2437087 & 0.2342646 & 0.5220267 \end{bmatrix} \begin{bmatrix} -0.0317283 \\ 0.0357917 \\ 0.0003020 \end{bmatrix} = \begin{bmatrix} 0.0012921860 \\ 0.0018032351 \\ 0.0008099162 \end{bmatrix}$$

for  $j = 3$

$$\begin{bmatrix} 0.2544837 & 0.2554949 & 0.4900214 \\ 0.2574339 & 0.2614577 & 0.4811084 \\ 0.2510994 & 0.2487950 & 0.5001056 \end{bmatrix} \begin{bmatrix} -0.0317283 \\ 0.0357917 \\ 0.0003020 \end{bmatrix} = \begin{bmatrix} 0.001218250 \\ 0.001335369 \\ 0.001088869 \end{bmatrix}$$

$\vdots$

for  $j = 14$

$$\begin{bmatrix} 0.2535647 & 0.2537065 & 0.4927289 \\ 0.2535647 & 0.2537065 & 0.4927289 \\ 0.2535647 & 0.2537065 & 0.4927289 \end{bmatrix} \begin{bmatrix} -0.0317283 \\ 0.0357917 \\ 0.0003020 \end{bmatrix} = \begin{bmatrix} 0.001184214 \\ 0.001184214 \\ 0.001184214 \end{bmatrix}$$

The expected short-runs are presented in the tables below

Table 4.3: **Expected Short-run returns**

State	t=1	t=2	t=3	t=4	t=5	t=6	t=7
Fall	0.0006972543	0.0012921860	0.001218250	0.001192996	0.001186434	0.001184774	0.001184355
Rise	0.0042972938	0.0018032351	0.001335369	0.001222175	0.001193780	0.001186626	0.001184822
Stable	-0.0001681168	0.0008099162	0.001088869	0.001160149	0.001178147	0.001182685	0.001183829

Table 4.4: **Expected Short-run returns**

State	t=8	t=9	t=10	t=11	t=12	t=13	t=14
Fall	0.001184250	0.001184223	0.001184216	0.001184215	0.001184214	0.001184214	0.001184214
Rise	0.001184367	0.001184253	0.001184224	0.001184217	0.001184215	0.001184214	0.001184214
Stable	0.001184117	0.001184190	0.001184208	0.001184213	0.001184214	0.001184214	0.001184214

From the table 4.3 and 4.4 above showing the expected short-run returns, we found the expected short-run returns from the first day ( $t = 1$ ) to the fourteenth day ( $t = 14$ ) when the returns become stable regardless of current state. It is seen that if the present state is a rise state, it is expected that a return greater than the overall average return (0.0013074) will be realized after only two days, but it will not be possible to realize a return greater than the average return if in the fall or stable state. This means that if an investor buys shares from Nestle Nigeria PLC, and it is in a rise state, he or she has to wait for two days without selling in order to realize return which is above the overall average daily return.

### **4.0.3 CAPM**

The risk premium of the CAPM was gotten from the relation  $(r_m - r_f)$  where  $r_m$  is the average return of the market for the year, estimated from daily closing prices of the Nigerian stock exchange All-share index. And  $r_f$  the average risk-free rate of return of the year, which is the yield on the government treasury bill which is relatively risk-free. Using equation (3.23), the  $\beta$  was found to be 0.1066, and using equation (3.21), the expected return was found to be 0.000336.

### **4.0.4 Comparison of Expected return of the Markov Chain Model and CAPM**

The expected long-run return value of 0.001184214 was obtained from the Markov chain model, the short-run returns for the three states are as shown in tables 4.3 and 4.4. The expected return for the CAPM was obtained to be 0.000336.

The study shows that the overall average of returns of the stock (0.0013074) will not be

realized in the long-run (i.e after 14 days regardless of present state). Also, depending on if the return is in a Fall, Rise, or Stable state, an investor will realize a return that is greater than the overall average return only if the returns are in a rise state after two days.

We also discovered that the expected return obtained from the CAPM will be realized in the short run after two days if the return is in the fall (0.0012921860) or stable state (0.0008099162), and after one day if the return is in the rise state (0.0042972938). Also regardless of present state of return the expected return of the CAPM will be realized in the long-run. This indicates that the stock was not overpriced by the Capital asset pricing model. Hence an investor will realize a positive return regardless of the present state in the long run.

Table 4.5: **Descriptive statistics of Returns**

Sample size	237
Expected long-run	0.001184214
Expected return (CAPM)	0.000336000
Average return	0.001307400

# CHAPTER 5

## CONCLUSION AND RECOMMENDATIONS

This work presents an application of the Markov chain model and the Capital asset pricing model on same data set of daily returns of Nestle Nigeria PLC listed on the Nigerian stock exchange.

The Markov chain model was used to estimate long and short-run expected returns. We find that regardless of present state of returns, the average returns will not be realized in the long-run, that is after fourteen days. Also depending on the present state, the average return will only be realized if the return is currently in a rise state after two days. The expected return from the CAPM was computed and compared to the long and short-run expected returns of the Markov chain model. The analysis suggests that the expected return of the CAPM is not over priced, so an investor will be better off investing on the stock.

For the first objective, the returns were arranged in order and divided into four parts. the first quartile  $Q_1$  and the third quartile  $Q_3$  were used as the threshold to determine movement between the states of the Markov chain. Returns less than  $Q_1$  represents the lower 25% of the data and were classified as the fall state, returns higher than  $Q_3$  represents the higher 25 percent of the data and were classified as being in rise states, while the returns between  $Q_1$  and  $Q_3$  inclusive represented the middle 50% of the data and were classified as stable returns. So the rise and fall states capture extreme movement of the returns to either side of the median.

Next, the successive days returns where categorized into three states (Fall, Rise, and



Stable)and then modeled using a three state markov chain model, where a day's return being in any state depends only on the return of the previous day.

## 5.1 Conclusions

The result from the study shows that the overall average of returns of the stock (0.0013074) will not be realized in the long-run (i.e after 14 days regardless of present state). Also, depending on if the return is in a Fall, Rise, or Stable state, an investor can realize a return that is greater than the overall average return only if the returns are in a rise state after two days.

Finally, it was noticed that the expected return gotten from the CAPM will be realized in the short run after two days if the return is in the fall or stable state, and after one day if the return is in the rise state. Also regardless of present state of return the expected return of the CAPM will be realized in the long-run. This hints that the stock was not overpriced by the Capital asset pricing model. Hence it indicates that an investor will realize a positive return regardless of the present state in the long run.

## 5.2 Recommendations

It is recommended that further research be carried out using a portfolio consisting of different stocks or on other instruments like gold and foreign exchange returns. Also since this work focuses on daily returns, more work can be done by analyzing smaller or larger interval of returns, like hourly, monthly or yearly returns of stock. Also there should be more work done to determine suitability of different threshold for different instruments, as the choice of threshold is usually decided by the researcher.

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